APPLIED PHYSICS

(Revised Syllabus NEP 2020 as per Mumbai University)

TEXTS

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INTERNATIONAL

Contents

SYLLABUS

Course Name: Applied Physics
Course Code: BSC102

Prerequisite:

1. Basic knowledge of optics and atomic structure, Wavefront and Huygens principle, reflection and refraction, Interference by division of wavefront, Refractive index of a material, Snell's law, Basics of vector algebra, partial differentiation concepts, Dual nature of radiation, Photoelectric effect, Matter waves, Davisson-Germer experiment. Intrinsic and extrinsic semiconductors, electrical resistivity and conductivity concepts

Chapter-1 Lasers 04 Hours

- Lasers: Spontaneous and stimulated emission, population inversion, pumping, active medium & active center, resonant cavity, coherence length and coherence time, Characteristics of lasers, He-Ne laser: construction and working. Fiber laser Construction and working Application : (i)Elementary knowledge of LiDAR (ii) Barcode reader (iii) Application of laser in metal work

Chapter-2 Fibre Optics 04 Hours

- Optical fibers: Critical angle, acceptance angle, acceptance cone, numerical aperture, total internal reflection and propagation of light, Types of optical fibers: Single mode & multimode, step index & graded index, attenuation, attenuation coefficient, factors affecting attenuation, Fibre Optic Communication System, Advantages of optical fiber communication, numerical

Chapter-3 Interference In Thin Film 04 Hours

- Interference in thin film of uniform thickness, conditions of maxima and minima for reflected system, Conditions for maxima and minima for wedge shaped film (qualitative), engineering applications – (i) Newton's rings for determination of unknown monochromatic wavelength and refractive Index of transparent liquid (ii) AntiReflecting Coating

Chapter-4 Electrodynamics 04 Hours

- Vector Calculus: Gradient, Divergence, Curl. Gauss's law, Amperes' circuital Law, Faraday's law, Divergence theorem , Stokes theorem Maxwell's equations in point form, Integral form and their significance(Cartesian coordinate only)

Chapter-5 Quantum Physics 06 Hours

- de Broglie hypothesis of matter waves, de Broglie wavelength for electron, Properties of matter waves, Wave function and probability density, mathematical conditions for wave function, problems on de Broglie wavelength, Need and significance of Schrödinger'sequations. Schrödinger's time independent and time dependent equations, Energy of a particle enclosed in a rigidbox and related numerical problems, Quantum mechanical tunneling, Principles of quantum computing: concept of Qubit.

Chapter-6 Basics Of Semiconduct or Physics 04 Hours

- Direct and Indirect Band Gap Semiconductors, Electrical Conductivity of Semiconductors, Drift Velocity, Mobility and Conductivity in Conductors Fermi- Dirac distribution function, Position of Fermi Level in Intrinsic and Extrinsic Semiconductors.

CHAPTER 1.

Laser

Laser: spontaneous emission and stimulated emission; metastable state, population inversion, types of pumping, resonant cavity, Einsteins's equations; Helium Neon laser; Nd:YAG laser; Semiconductor laser, Applications of laser- Holography

1.1 Introduction to LASER

Laser is the device that produces monochromatic, coherent, and highly directional light. Laser is an acronym for light amplification by stimulated emission of radiation. Hence, the process of stimulated emission causes amplification or increases the intensity of light. Invention of laser has the background of the work published by Max Planck in 1900 that light is a form of electromagnetic radiation, the theory considered as the mark of modern physics.

The principle of laser was first known in 1917, when physicist Albert Einstein in 1917, in the course of calculations on the equilibrium of a gas, discovered that there must be spontaneous and stimulated types of emission. Applying the idea of amplification by means of stimulated emission, in 1953 Charles Townes and his collaborators at Columbia University constructed a device and christened it 'MASER', an acronym for microwave amplification by stimulated emission of radiation. This device soon found application in microwave communication system.

Schawlow and Townes in 1958 suggested that the principle of the maser can be extended to the optical regime. In 1960 the American physicist, Theodore Maiman invented the first laser, a solid state laser, using a lasing medium of ruby. Later in 1960 the Iranian physicist, Ali Javan constructed first gas laser using helium and neon.

In 1962 Robert Hall demonstrated first semiconductor diode laser. Indian electrical engineer Kumar Patel invented CO_2 laser in 1964 at Bell laboratory. In 1994, Markos and his coworkers at Bell laboratory developed Nd-YAG laser. A few other laser devices are dye lasers, liquid lasers, chemical lasers, photonic crystal lasers etc. Photonic crystal lasers are based on nano-structures that provide the mode confinement.

Recall Bohr's model of an atom. According to Bohr's third postulate if an atom at higher energy state makes a transition to lower energy state it emits a photon of energy $h\vartheta$ equal to the energy difference between the two states.

In the process of emission of light from conventional sources some of the atoms may spontaneously move from say E_5 to E_4 state, or E_5 to E_3, or E_5 to E_2 while others may undergo

transition from E_4 to any lower state and so on. As a result the emitted photons have different energies or wavelengths. Such a radiation is non- monochromatic as shown in Fig. (1.1).

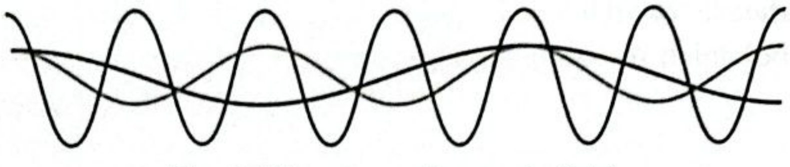


Fig: 1.1 Non-monochromatic light

Further, even if all the atoms spontaneously undergo transition say E_2 to E_1, it is not possible for all the atoms to jump down at the same time. Thus though the radiation emitted is monochromatic it is not in the same phase. It is incoherent radiation as shown in Fig. (1.2).

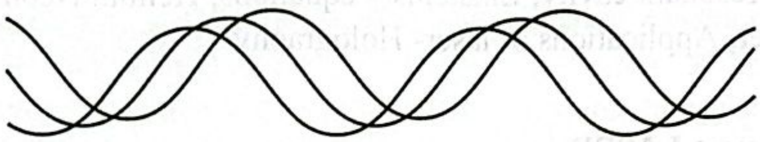


Fig: 1.2 Incoherent light

Also, since photons are emitted in all directions the radiation is not unidirectional. The radiation from the ordinary source is therefore as shown in Fig.(1.3).

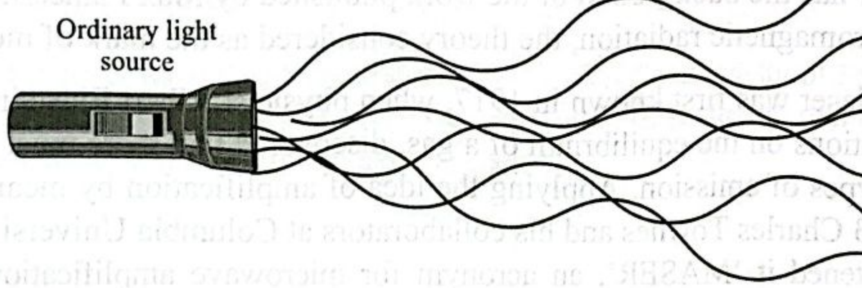


Fig: 1.3 Light emitted by ordinary source

On the other hand, the wavelength (colour) of laser light is extremely pure (monochromatic) compared to other sources of light. Also, all the photons (energy) that make up the laser beam maintain fixed phase relationship (coherence) with respect to one another.

Light produced by a laser can travel over a long distance with a very little divergence. It is highly directional and can be focused to a very small spot which is very bright. Thus the intense, monochromatic, coherent, and collimated beam of light finds a variety of exciting applications in various fields.

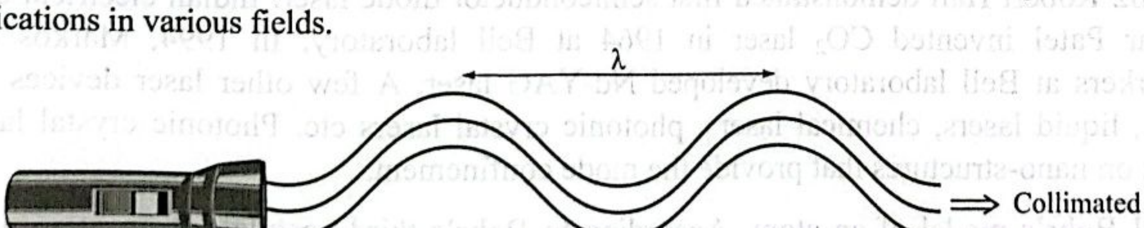


Fig: 1.4 Light emitted by LASER source

The lasers emit radiation of wavelengths ranging from the microwave region and infrared to visible, ultraviolate, to the soft X-ray spectral region. They produce power that ranges from

nanowatts to billion trillion watts (10^{21} W) for very short bursts. They produce the burst of light of very short duration of five million- billionths of a second (5×10^{-15} sec).

1.2 Difference Between Ordinary light and Laser light

Characteristics	Ordinary Light	LASER
Coherence	Incoherent source of light.	Highly coherent (maintains fixed phase relationship).
Wavelength	Polychromatic source of light.	Monochromatic source of light.
Direction of travel	Radiation is not unidirectional.	Unidirectional light.
Polarization	Light is not polarized.	Highly polarized (direction of oscillating electric field perpendicular to direction of travel).
Intensity	Has comparatively less intensity than laser.	It is a highly intense beam.

1.3 Quantum Processes

The lasers use very specific materials to produce their beams and the radiation emitted is often very pure and of a very specific wavelength. Lasers operate on the principle of quantum theory of radiation. In order to understand how a laser works it is imperative to recount the quantum process that occurs in the material when radiation is incident on it. For sake of explanation let us restrict initially only to two energy states of the atom. Let E_1 be the ground state and E_2 the excited state.

(a) Photon absorption (Stimulated absorption): If an atom is in the lowest possible energy state i.e. ground state E_1, it will absorb a photon of energy $h\vartheta = E_2 - E_1$ from the incident radiation and raises itself to the excited state E_1. This is known as stimulated absorption or simply absorption as shown in Fig. 1.5.

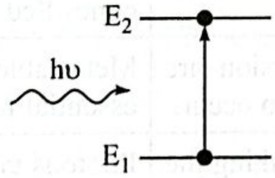


Fig: 1.5 Schematic diagram of atomic absorption

Symbolically, $atom + photon \longrightarrow atom^*$ (1.1)

(b) Spontaneous emission: The inherent tendency of an atom is to be in the lowest possible state i.e ground state. Therefore an atom in the higher energy (excited) state E_2 seeks to attain ground state E_1 by emitting a photon of energy $h\vartheta = E_2 - E_1$ on its own without any external influence. This process of de-excitation is termed as spontaneous emission as shown in Fig. 1.6.

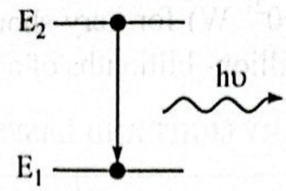


Fig: 1.6 Schematic diagram of atomic spontaneous emission

Symbolically, $atom^* \longrightarrow atom + photon$ (1.2)

(c) Stimulated emission: Before an atom in the excited state undergoes spontaneous emission, if a photon having energy equal to the energy difference between the two levels interacts with it and induces it to make transition from its excited state to ground state with emission of a photon, then that process of de-excitation is called stimulated emission which gives two photons, one is the incident photon and second one is emitted due to downward transition. These two photons are identical with respect to energy, wavelength (or frequency), phase, state of polarization and direction of propagation. Thus the emitted radiation is amplified, unidirectional, coherent, narrow and monochromatic. This is the working principle of a Laser. A schematic representation of it is shown in Fig. 1.7.

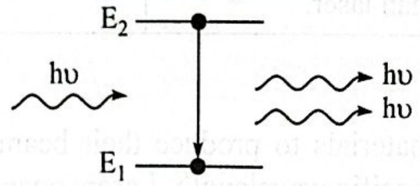


Fig: 1.7 Schematic diagram of atomic stimulated emission

Symbolically, $atom^* + photon \longrightarrow atom + 2\ photon$ (1.3)

1.4 Difference Between Spontaneous and Stimulated Emission

Spontaneous Emission	Stimulated Emission
An excited molecular state relaxes to lower energy state naturally.	An excited molecular state relaxes to lower energy state when stimulated by another photon.
It is an arbitrary process and cannot be controlled from outside.	It is not an arbitrary process and can be controlled from outside.
Metastable state and population inversion are not required for spontaneous emission to occur.	Metastable state and population inversion is essential for simulated emission to occur.
Photons are emitted in all directions making the light non-directional.	Photons emitted have the same direction as that of stimulating photon and hence light produced is directional.
Photons emitted are of different frequencies and hence light is not monochromatic.	Difference in the frequency of the emitted photons are very small making the light almost monochromatic.
Emitted photons do not have a phase relationship with each other. Thus the light produced is incoherent.	All emitted photons are in phase with each other. Thus the light produced is coherent.
Amplification of light does not occur due to multiplication of photons.	Light amplification occurs due to multiplication of photons.

1.5 EINSTEIN'S EQUATIONS

Laser beam is obtained only if the rate of stimulated emission exceeds the rate of spontaneous emission. The rate of absorption of incident photons depends on the number of atoms in the ground state and the energy density of incident photons. Mathematically it can be given by,

$$r_{\text{absorption}} = B_{12}\, N_1 Q \tag{1.4}$$

where B_{12} is the proportionality constant called Einstein's coefficient, N_1 the number of atoms at the ground state, and Q the energy density (i.e. the number of photons that have exact energy $h\vartheta$ required for transition from E_1 to E_2).

The rate of stimulated emission depends on the number of atoms at the upper state and can be given by

$$r_{\text{stimulated}} = B_{21}\, N_2 Q \tag{1.5}$$

where B_{21} is the Einstein's coefficient, N_2 the number of atoms at the upper energy state, and Q the energy density (i.e. the number of photons that have exact energy $h\vartheta = E_2 - E_1$).

Since spontaneous emission is a natural process the rate of spontaneous emission is independent of the incident energy density but depends only on the number of atoms at the upper state. Thus

$$r_{\text{spontaneous}} = A_{21}\, N_2 \tag{1.6}$$

where A_{21} is the Einstein's coefficient, and N_2 the number of atoms at upper state available to emit photons. The processes are schematically shown in Fig. 1.8.

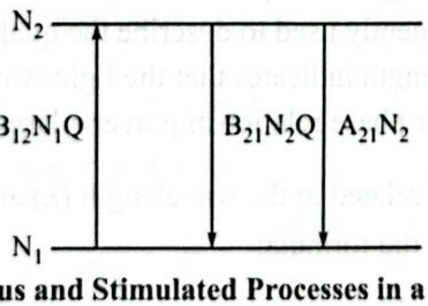


Fig. 1.8: Spontaneous and Stimulated Processes in a Two Level System

1.6 COHERENCE LENGTH AND COHERENCE TIME:

Coherence refers to the extent to which electromagnetic radiation retains a closely consistent phase connection, covering both temporal and spatial dimensions. The time for which coherence remains intact is referred to as coherence time. And the length that a signal could travel through a vacuum within this timeframe is denoted as the coherence length.

For example, in the case of light-emitting diodes, the coherence time typically falls within half a picosecond, while the associated coherence length is approximately 15 microns. Conversely, a basic laser might exhibit a coherence time of roughly half a nanosecond, accompanied by a coherence length of around 15 centimeters.

In the case of a high-quality laser with a narrow linewidth, the coherence time could extend to about a microsecond, while the coherence length could potentially stretch up to 200 meters.

The range of signal variation (referred to as linewidth) and the coherence length, as well as coherence time, are inversely proportional.

$$Length_{coherence} = c \times Time_{coherence} = \frac{\lambda^2}{\Delta\lambda} \tag{1.7}$$

distance = speed x time

$$Lcoh = c \times \tau_{coh} = \frac{\lambda^2}{\Delta\lambda} \tag{1.8}$$

Where Δλ is the linewidth and λ is the center wavelength

L_{coh} is Coherence Length

τ_{coh} is Coherence Time

c is Speed of Light

1.6.1 Coherence Length

Coherence length is a measure of the distance over which a wave maintains a consistent phase relationship. In other words, it is the distance over which a wave retains its characteristic waveform before significant phase changes or wave interference occurs. In optics, coherence length is frequently used to describe the quality of light sources, such as lasers. A longer coherence length indicates that the light waves from the source are more synchronized and maintain their phase relationship over a larger distance.

The coherence length (L_{coh}) is related to the wavelength (λ) and the degree of spectral bandwidth (Δλ) of the wave by the formula:

$$L_{coh} = \frac{\lambda^2}{\Delta\lambda} \tag{1.8}$$

where a smaller spectral bandwidth or a longer wavelength results in a longer coherence length. Coherence length is important in various applications, including interferometry, holography, and optical communication systems.

1.6.2 Coherence Time

Coherence time is a measure of the time duration over which a wave maintains its coherence or phase relationship. In quantum mechanics, coherence time is often associated with the duration in which a quantum system can remain in a superposition of states before decoherence sets in and the quantum properties are lost due to interactions with the environment. Coherence time is a critical factor in quantum computing and quantum

information processing, where maintaining the delicate quantum states is essential for performing complex computations.

The coherence time (τ_{coh}) can be related to the spectral linewidth (Δv) of the wave by the formula:

$$\tau_{coh} = \frac{1}{\lambda} \tag{1.9}$$

Similar to coherence length, a smaller spectral linewidth leads to a longer coherence time. Coherence time is a crucial parameter in fields such as quantum optics, nuclear magnetic resonance (NMR) spectroscopy, and quantum cryptography.

1.7 Metastable State

An atom can stay in the excited state for a very short period of 10^{-8} second as shown in Fig. 1.9. After that they undergo transition to ground state naturally (spontaneously). The spontaneously emitted photons have no preferred direction and they do not have phase relations with each other. The light generated is therefore incoherent. (Repeated) It is not however necessary that the atom is always de-excited to ground state. It can go to an intermediate state, called metastable state with non-radiatiative transition, where it stays for a much longer period than in the upper (excited) level and then come down to the ground state.

Since the atoms can reside in the metastable state for longer time (about 10^{-4} to 10^{-3} second) they accumulate in this state and their de-excitation to ground state gives out coherent radiation. In fact the metastable state is necessary for lasing action.

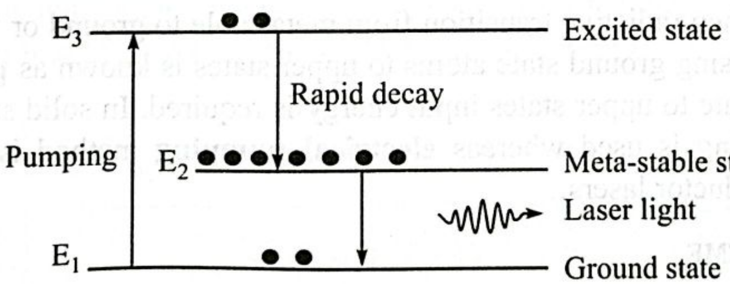


Fig. 1.9: Metastable state for Laser action

1.8 Population Inversion

Usually the number of atoms in the ground state is more than that in higher states. Boltzmann principle, which is the fundamental law of thermodynamics, states that at thermal equilibrium the relative population between any two energy levels is given by,

$$\frac{N_2}{N_1} = exp\left(-\frac{E_2 - E_1}{kT}\right) \tag{1.10}$$

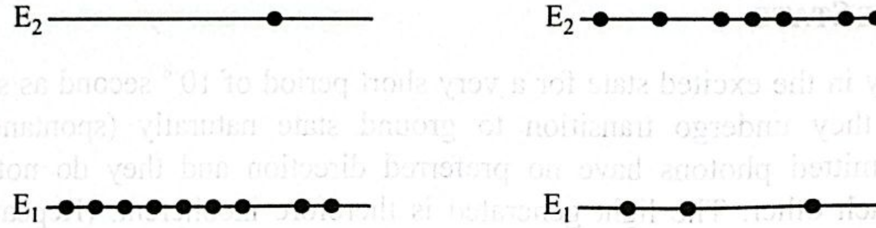
where, N_1 and N_2 are the populations of the lower and upper energy states respectively, T the equilibrium temperature, and k the Boltzmann constant. Since $E_2 - E_1 = h\vartheta$, Eqn. (1.11) can be written as

$$\frac{N_2}{N_1} = exp\left(-\frac{h\vartheta}{kT}\right) \tag{1.11}$$

But for stimulated emission to occur there must be more number of atoms in the metastable (upper lasing level) state. If $E_1$is the ground state and E_2 represents metastable state then always the number of atoms N_2 in the metastable state should be far greater than number of atoms N_1 in the ground state (lower lasing level).

Thus the condition $N_2 > N_1$, called population inversion is essential for lasing.

A medium in which population inversion is achieved is called an **active medium** also called a **gain medium.** A laser is produced by providing energy to an active medium which is the substance that produces the light in the precise wavelength of the laser. The active medium may be solid, liquid, or gas. The atoms in the active medium responsible for stimulated emission and light amplification are called **active centers.**



(a) Normal Population **(b) Population Inversion**

Fig. 1.10

1.9 PUMPING

To achieve population inversion it is necessary that the atoms in the ground state be continuously transferred to excited state from where they make non-radiative transition to the metastable state and then radiative transition from metastable to ground or lower lasing level. The mechanism of raising ground state atoms to upper states is known as **pumping.** To raise atoms from ground state to upper states input energy is required. In solid state lasers and dye lasers **optical pumping** is used whereas **electrical pumping** method is employed in gas lasers and in semiconductor lasers.

1.9.1 PUMPING SCHEME

(a) Two-level Pumping Scheme: Suppose there are only two levels, a ground state E_1 and the metastable state E_1. Pumping raises the ground state atoms to this metastable state. As the population inversion is attained the laser would lase and there would be more atoms in the ground state. Thus to maintain population inversion atoms must be energetically pumped to metastable state E_2 from the outside. Although the model of two-level pumping is theoretically sufficient to explain properties of laser light, actually building a laser with two energy level scheme becomes somewhat more complicated.

(b) Three-level Pumping Scheme: In a three-level atom system the ground state atoms are pumped to uppermost energy level E_3 when light of energy $h\vartheta = E_3 - E_1$ is incident on them. Level E_3 is unstable. The excited atoms at this level make spontaneous downward transition before 10^{-8} seconds either to the ground state E_1 or to an intermediate state E_1. The spontaneous transition $E_3 \longrightarrow E_2$ is more probable than E_3

$\rightarrow E_1$. The transition $E_3 \longrightarrow E_2$ is non-optical and the energy difference E_3 - E_2 is transferred to the lattice. The number of atoms in the temporary stable E_2 state increases. When more than half of the ground state atoms gather at E_2, a photon of energy $h\vartheta = E_2 - E_1$ released in the spontaneous transition, which is a rare event, causes stimulated emission.

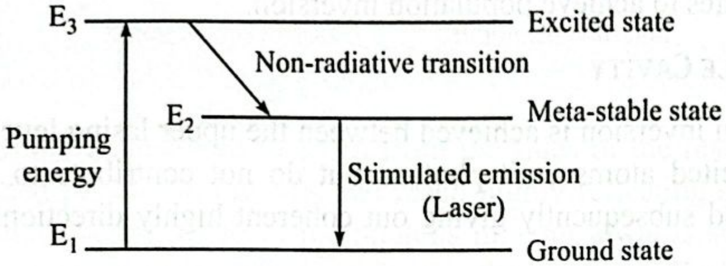


Fig. 1.11 Three level pumping scheme

In three-level laser system, metastable state is the upper lasing level and ground state is the lower lasing level. To pump more than half of the ground state atoms to the upper lasing level (metastable state) a very high energy is required.

(c) **Four-level Pumping Scheme:** In four-level pumping scheme ground state E_1 is not the lower lasing level. Atoms are pumped from ground state to the uppermost level E_4. There is non- radiative rapid decay from E_4 to energy level E_3 which is the metastable state.

The laser transition from metastable state terminates at an unstable intermediate state E_2 rather than at the ground state. The intermediate state decays non-radiatively very fast to the ground state. Since population inversion is to be achieved between E_3 and E_2 a modest amount of pumping is sufficient to populate the metastable state.

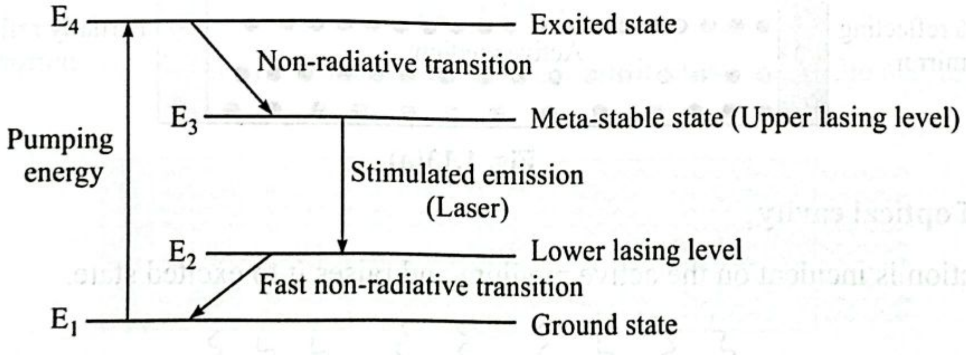


Fig. 1.12 Four Level Pumping Schemes

1.9.2 PUMPING TECHNIQUES

The most common methods of pumping are described below.

(a) **Optical Pumping:** This is the efficient method for solid state lasers as they have broad absorption bands. This method uses photons to excite the atoms to the higher energy state, generally, by means of flash lamp. The incident photons from the flash lamp are used by the atoms of the laser medium to excite to the higher energy states to achieve population inversion.

(b) **Electrical Pumping:** This type of pumping can be used in lasers where the lasing activity and electrical discharge do not interfere with each other. It is mainly used in gas laser, dye laser and semiconductor laser. In gas laser, high voltage ionizes the gas. Electrons in the discharge tube are accelerated by electric field which in turn collide with the atoms, ions or molecules in the active medium and are excited to higher energy states to achieve population inversion.

1.10 Resonance Cavity

When population inversion is achieved between the upper lasing level and lower lasing level, most of the excited atoms emit photons but do not contribute to the overall output. For amplification and subsequently giving out coherent highly directional intense light beam is necessary.

An optical resonator known as Fabry-Perot resonator that consists of two mirrors facing each other between which an active medium is placed provides positive feedback to the medium and turns the system into a laser.

The mirrors are perfectly parallel to each other and perpendicular to the axis of the active medium. The mirror at one end is made 100% reflecting while the other one is semitransparent as shown in Fig. 1.13a.

Such an optical configuration of two plane parallel mirrors with a plane parallel slab of air between them was invented and used by Marie Fabry and Jean Perot in their study of interference effect produced by multiple reflections.

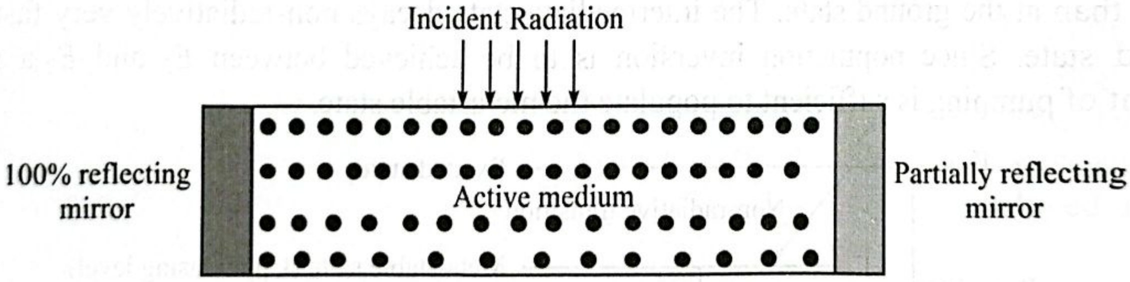


Fig. 1.13(a)

Working of optical cavity

(a) Radiation is incident on the active medium and raises it to excited state.

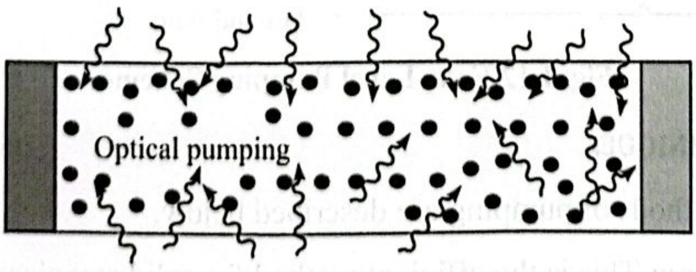


Fig. 1.13(b)

(b) Excited atoms emit photons in all directions.

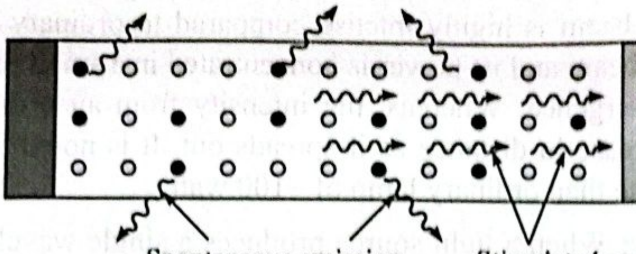


Fig: 1.13 (c)

(c) The off-axis photons are undesirable pass out from the sides of the resonator and are lost. Those are emitted parallel to the axis and reach highly reflecting mirror, bounce back into the medium whereas some of the photons moving towards semitransparent mirror pass out and rest of them reflect back into the medium. Each of the axial photons in its back and forth journey triggers one excited atom to undergo stimulated emission.

Each stimulated transition gives out two photons. The process continues and number of photons double at each step. As a result number of photons having same frequency, phase, and the direction goes on increasing enormously. In fact the resonance cavity acts as an oscillator by returning some of the photons back into medium where all the photons are in the same phase. Thus the light wave of large amplitude is built up between the mirrors.

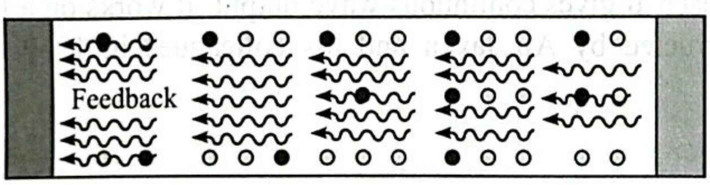


Fig. 1.13(d)

(d) Finally when the beam of sufficient strength is built it burst out of the semitransparent mirror.

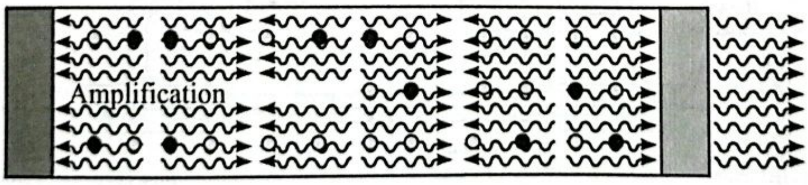


Fig. 1.13 (e)

1.11 CHARACTERISTICS OF LASER

The most salient features of a laser beam are:

(a) Directionality: Radiation from a laser source travels in a single direction only with minimal divergence. Owing to this, it can be transmitted over long distance. As the active medium is placed in a resonant cavity its gets reflected back and forth inside the cavity. The light that travels parallel to the axis only is emitted as laser beam and rest of the light is eliminated.

(b) Intensity: A laser beam is highly intense compared to ordinary light. Laser radiates light into a narrow beam and its power is concentrated in a small region of space owing to its very less divergence. Whereas, the intensity from an ordinary source of light decreases with increase in distance as it spreads out. It is noted that a laser of 1 watt appears more intense than ordinary lamp of ~100 watt.

(c) Monochromaticity: When a light source produces a single wavelength it is said to be monochromatic. In reality this is not possible. Thus a light source is characterized not by a single wavelength but by spread in wavelength about the central wavelength. The wavelength spreads over a range of ~100Å to 1000Å for ordinary light, whereas for laser it is less than 10Å.

(d) Coherence: It is the most prominent feature which distinguishes laser from ordinary light. Laser has long (several kilometers) trains of wave characterized by high degree of space and time coherence. Two waves are said to be coherent if they have zero or constant phase difference between them. It is feasible to obtain interference pattern from two independent laser sources which is impossible in case of ordinary light. As an ordinary light source is a jumble of short wave trains that reinforce in a random manner resulting in an incoherent source of light. Coherence length for a sodium lamp is ~0.3mm whereas for He-Ne laser it is ~100m.

1.12 HELIUM-NEON LASER

He-Ne laser is a gas laser. It gives continuous wave output. It works on a four level pumping scheme. It was constructed by Ali Javan and his colleagues in 1960 at Bell Telephone Laboratories.

Construction

A mixture of helium and neon in the ratio 10:1 at a low pressure about 1 torr (equivalent to 1mm of mercury) is placed in a quartz tube about 100 cm long and 1.5 cm internal diameter as shown in Fig. 1.14.

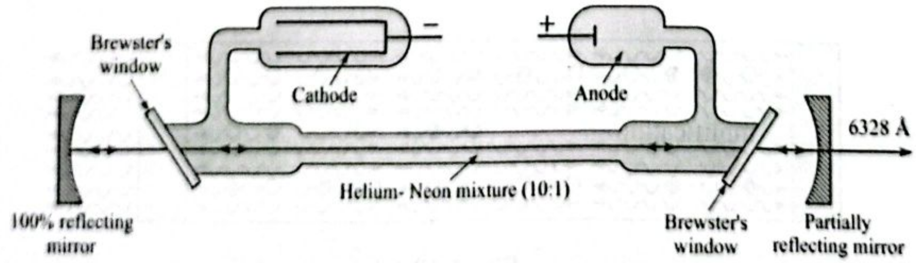


Fig. 1.14 Helium Neon Laser Tube

To prevent the loss of the beam Brewster-angle windows are attached to each end of the discharge tube. When Brewster-angle windows are used, the beam is automatically polarized in a single transverse direction.

According to Brewster's law, if an unpolarized beam is incident at an angle,

$\theta = \theta_p = tan^{-1}\left(\frac{n_2}{n_1}\right)$, then the reflected beam will be plane polarized. Here θ_p is known as polarizing angle or Brewster's angle, n_1 the refractive index of air (or gas) and n_2 the refractive index of window glass.

A high voltage power supply provides current to the system. Two concave mirrors facing each other one finely reflecting and the other semitransparent are fixed at two ends on the axis of the tube forms a Fabry-Perot resonator.

Working

In He-Ne laser, neon atoms are the active centers. Helium atoms help neon atoms gain sufficient energy and raise them to the higher energy states.

When an electric discharge is passed through the gas the electrons collide with helium and neon atoms. Helium atoms are lighter and therefore get excited from its ground state F_1 to higher energy state F_1. The F_2 state is metastable and its energy is 20.61 eV.

Further the transition $F_2 \rightarrow F_1$ is not allowed. The helium atoms in F_2 state collide with ground state neon atoms. Helium atoms return to ground state by imparting their energy 20.61 eV to neon atoms so that neon atoms are raised to higher energy state E_4.

Also, the kinetic energy with which helium atoms collide with neon atoms provides additional 0.05 eV energy to the neon atoms. This way the neon atoms which are the active centers are pumped to E_4 state which is metastable. The energy of E_4 state is thus 20.66 eV.

Population inversion is achieved between upper lasing level E_4 and lower lasing level E_3. Lasing takes place in a transition $E_4 \rightarrow E_3$. The wavelength of emitted photon is 6328 Å. The Fabry-Perot resonator builds a highly coherent, directional and intense laser beam.

The transition from E_3 to E_2 is spontaneous one and yields only incoherent light. The atoms in the E_2 state come to the ground state E_1 by losing their energy in collision with the tube walls. Since the excitation of helium and neon atoms due to electron impacts takes place incessantly, a He-Ne laser operates continuously and hence emits continuous wave.

Energy Level Diagram

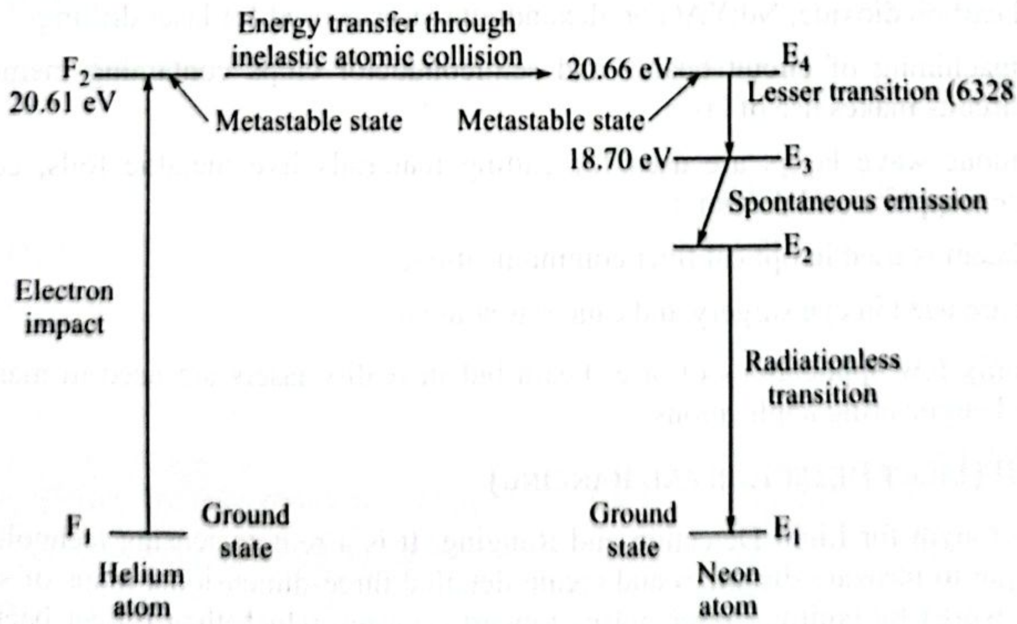


Fig: 1.15 Energy Level Diagram of Helium Neon Laser

The most often used laser transition in neon is red color laser operating at 6328 Å. It is the easiest wavelength to produce since it has the highest gain. The other visible wavelengths that can be produced are green laser (5435 Å), yellow laser (5940 Å), and orange laser (6120 Å). In the near infrared there are transitions in the 11500 Å (1.15μm), 15230 Å (1.523 μm), and 33900 Å (3.39 μm).

- **Advantages:** It emits more directional and more monochromatic light. It is a continuous wave laser. It can operate continuously without the need for cooling.
- **Disadvantage:** It has a small output.
- **Applications:** It is used to read bar codes, in scanners and printers and in holography.

1.13 Applications of Laser

Laser finds applications in the field of scientific research, military operations, industry, communication, medicine, entertainment, etc.

1. The initial famous application of laser was made on the lunar ranging experiment of Apollo II Mission of 1969, when an array of retro reflectors was mounted on the surface of the moon and pulses from a ruby laser were sent from the earth. The reflected beams were received by suitable detectors and by measuring the time taken by the pulses in going from the earth to the moon and back, the distance of the moon from the earth was calculated to an accuracy of 15 cm.
2. In military operations the laser range finders which work on the principle of a radar use neodymium and carbon dioxide lasers have become a standard item to know the distance of the enemy tank and other targets.
3. In naval applications, instead of ultrasonic waves lasers are used as source of underwater transmission. For efficient ranging and detection frequency doubled Nd:YAG laser or an argon gas laser or a Raman shifted xenon chloride laser is used.
4. A missile can be guided and controlled by an infrared beam emitted from a laser.
5. Pulsed carbon dioxide, Nd:YAG or alexandraite laser is used for laser drilling.
6. Micromachining of circuit boards and semiconductor chips containing memory and logic circuits makes use of laser.
7. Continuous wave lasers are used for cutting materials like metallic foils, ceramics, graphite, sapphire and diamond.
8. Laser beam is used in optical fiber communication.
9. Lasers are used in eye surgery and cancer treatment.

Above are only few applications of laser beam but in reality lasers are used in many other scientific and engineering applications.

1.14 LiDAR (Light Detection and Ranging)

LiDAR is acronym for Light Detection and Ranging. It is a remote sensing technology that uses laser light to measure distances and create detailed three-dimensional maps of surfaces. The system works by emitting laser pulses toward a target, which then reflect back to the sensor. By measuring the time it takes for the pulses to return (time-of-flight), the system

calculates the distance to the target. These measurements, combined with precise GPS and orientation data, form a dense point cloud of 3D coordinates.

$$\text{The Distance of the object} = \text{Speed of Light} \times \frac{\text{Time of Flight}}{2}$$

A LiDAR system consists of a laser source, scanner and optics, a photodetector, a GPS receiver, an IMU (Inertial Measurement Unit), and a computer system for data processing. Aero planes and helicopters are the most popular platforms for collecting LiDAR data overlarge areas. There are two types of LiDAR: topographic and bathymetric. Topographic LiDAR typically uses a near-infrared laser to map the land, while bathymetric LiDAR uses water-penetrating green light to measure the elevation of the seafloor and riverbed. Its advantages include high accuracy, longer range, and better resolution, the ability to penetrate vegetation, rapid data collection, and versatility in mounting on various platforms.

LiDAR technology is used to create detailed topographic maps in a variety of applications, including mapping and surveying. Environmental monitoring, forestry, agriculture, urban planning, archaeology, and self-navigating vehicles for obstacle detection.

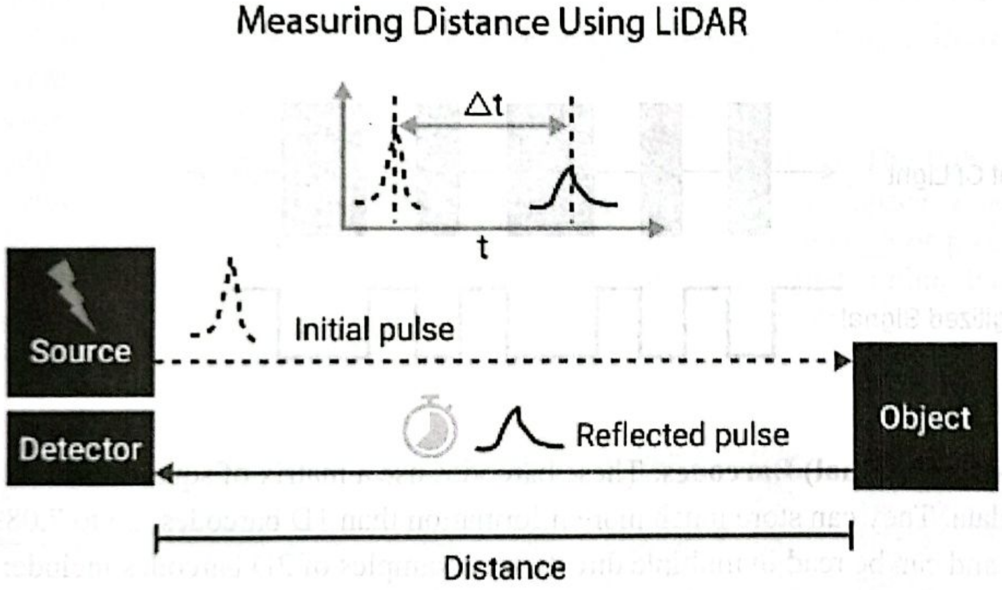


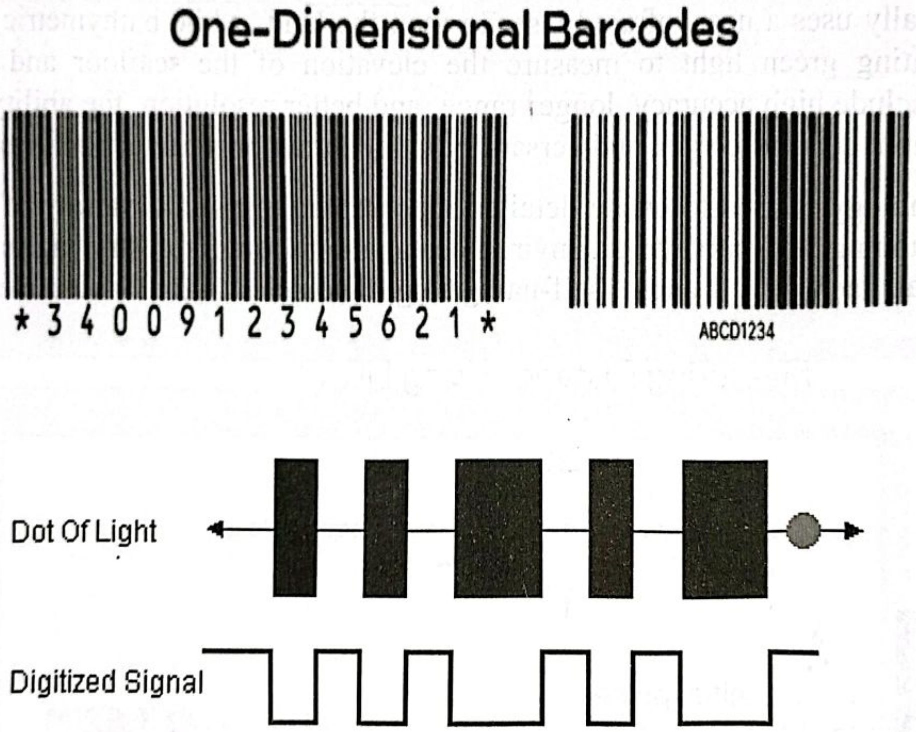
Fig: 1.16 Distance measurement by LiDAR

1.15 BARCODE

A barcode is a unique pattern that can be read by machines to identify and track products, items, or information. Invented by Bernard Silver and N. Joseph Woodland in the 1940s, barcodes have revolutionized shopping, inventory management, and information access. They're used in various industries, including retail and healthcare, to streamline processes, reduce errors, and increase efficiency. Commonly referred to as UPC, QR code, EAN, and Data Matrix. These barcode commonly referred as,

a) **1-D (One-Dimensional) Barcodes:** These barcodes consist of a single row of lines and spaces that represent data. They are read from left to right and can store a limited amount of information, typically up to 20 characters. Examples of 1D barcodes include:

- UPC (Universal Product Code)
- EAN (European Article Number)
- Code 128
- Code 39



b) 2-D (Two-Dimensional) Barcodes: These barcodes use a matrix of squares or dots to represent data. They can store much more information than 1D barcodes, up to 7,089 characters, and can be read in multiple directions. Examples of 2D barcodes include:

- QR Code (Quick Response Code)
- Data Matrix
- PDF417
- Aztec Code

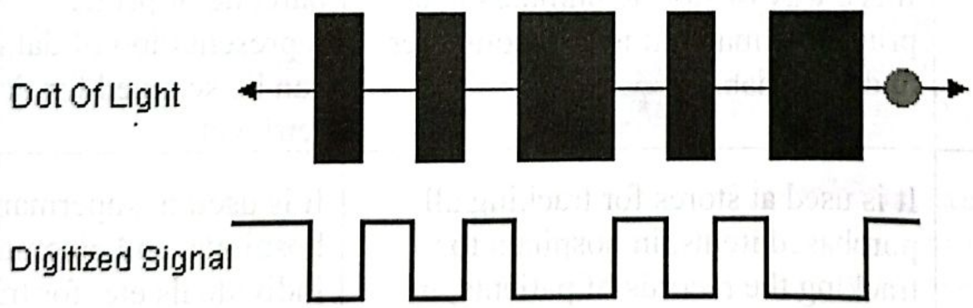
2D barcodes are often used for more complex applications, such as storing URLs, contact information, or even small images.

Two-Dimensional Barcodes

1.15.1 Barcode Reader

A barcode reader or barcode scanner is an optical scanner that can read printed barcodes and send the data they contain to computer.
It consists of a light source, typically a laser or LED, to illuminate the barcode, and a sensor to capture the reflected light. The reflected light is converted into an electrical signal, which is then decoded into digital data. This data is transmitted to a computer or system for processing.

Process of reading barcodes:
Barcode can be read by dragging the barcode tip across its bars. The light was absorbed by the dark bars and the lights were reflected back by the white space. Laser scanners, pan readers, camera-based readers, and CCD readers are the technology or process that helps in the process of reading a barcode. It is also important to note that reading barcodes also faces some challenges. They can be such as bleed in ink, colour contrast that is insufficient, peeling of the barcode labels and many more.



Types of Barcode Readers:-

1) Pen Scanners: Shaped like a pen, these devices are swiped across the barcode to read the data.
2) Handheld Scanners: These are portable and manually operated devices, commonly used in retail settings for scanning product barcodes at checkout counters.

3) Laser Scanners: can quickly scan barcodes from various distances and angles
4) Mobile Computers: Portable devices with integrated barcode scanners, often used in logistics and fieldwork for tracking inventory and managing assets on the go.
5) 2D Scanners: These are capable of reading both 1D and 2D barcodes, such as QR codes.
6) Fixed-Mount Scanners: These stationary scanners are used in high-volume environments like warehouses, where products or packages pass by the scanner.
7) CCD readers: Charge-coupled device (CCD) readers use an array of hundreds of tiny light sensors lined up in a row in the head of the reader. Each sensor measures the intensity of the light immediately in front of it.

1.15.2 DIFFERENCE BETWEEN 1-D BARCODE AND 2-D QR CODE

S.No.	1-D Barcode	2-D QR code
1	It was introduced in the year 1952.	It was introduced in the year 1994.
2	Its developer is Norman Joseph Woodland.	Its developer is Masahiro Hara.
3	There are 2 types of Barcode, 1 dimensional and 2 dimensional.	There is only 1 type of QR Code.
4	It is a way of storing numbers in a printed format that is also computer understandable.	It is a type of 2 dimensional barcode or printed representation of data that can be scanned for data retrieval.
5	It is used at stores for tracking all purchased items, in hospitals for tracking the records of patients, in rental cars business and tracking of airline luggage, mails and nuclear waste.	It is used in supermarkets, hospitals, and cinema or by individuals etc. for transfer of data (sharing contacts, photos, videos and other documents.
6	The storage capacity is greater than 100 bytes.	The storage capacity is 2 kilo bytes.

1.16 APPLICATION OF LASER IN METAL WORK

Lasers are widely used in metalworking for their precision and efficiency. Key applications include:

- **Laser Cutting:** Laser cutting uses a high-power laser beam to precisely cut metals such as steel, aluminum, and copper, resulting in less waste and clean edges. It is extensively used in the automotive, aerospace, electronics, and manufacturing industries to create parts and intricate designs.
- **Laser welding**: Melts metal pieces together with a laser beam, resulting in high speed, deep penetration, low heat impact, and the ability to weld difficult-to-reach areas. It is used in the automotive, aerospace, medical device, and electronics industries to produce strong, precise welds in materials such as stainless steel, aluminum, and titanium.
- **Surface Treatment:** The use of a laser beam to modify metal surfaces improves hardness, wear resistance, and corrosion resistance. Laser hardening, alloying, and texturing techniques are used on metals such as steel, titanium, and aluminum. It is used in the automotive, tooling, and electronics industries to improve component durability and performance.
- **Laser drilling**: To create precise holes in metals, material is melted and vaporized. It is known for its high precision, ability to create intricate holes, and compatibility with metals such as stainless steel, titanium, and nickel alloys. Common applications include aerospace (cooling holes in turbine blades), electronics (microvans on circuit boards), and medical devices.
- **Laser additive manufacturing Laser 3D Printing:** The process of layering metal powder or wire with a laser to create precise, complex objects. It is known for its high precision and low material waste when used with metals such as titanium and stainless steel. It is used in aerospace for lightweight structures, medical for customized implants, and automotive for rapid prototyping and high-performance parts.
- **Laser Engraving and Marking:** Using a focused laser beam, create detailed designs or information on metal surfaces by removing material or changing color. It has high precision, durability, and is applicable to metals such as steel, aluminum, and titanium, as well as plastics and ceramics. Applications include product branding, personalization, serial numbers, and decorative markings in industries such as electronics, automotive, and consumer goods.

Solved Problems

1. White light has a frequency range from 0.4 x 10^{15} Hz to 0.7 x 10^{15} Hz. Find the coherence time and coherence length for it.

Solution: Given: frequency $\upsilon_1 = 0.4 \times 10^{15}$ Hz, $\upsilon_2 = 0.7 \times 10^{15}$ Hz

Formula: $\tau_{coh} = \frac{1}{\Delta\upsilon}$ where $\Delta\upsilon$ is the frequency bandwidth

First, let's calculate the frequency bandwidth ($\Delta\upsilon$)

Time = $\frac{1}{freq}$; Dist = Speed x time

$$\Delta\upsilon = \upsilon_1 - \upsilon_2 = (0.7 - 0.4) \times 10^{15}\ Hz = 0.3 \times 10^{15} Hz$$

Now, we can calculate the coherence time (τ_{coh})

$$\tau_{coh} = \frac{1}{\Delta\upsilon} = \frac{1}{0.3 \times 10^{15}} = 3.33 \times 10^{15} sec$$

-15

Next, to find the coherence length (L), we use the formula:

$$Lcoh = c \times \tau_{coh}$$

$$Lcoh = 3 \times 10^{8} \times 3.33 \times 10^{15} = 0.001\ km\ or\ 1mm$$

So, the coherence time for white light is approximately 3.33 x 10^{-15} seconds, and the coherence length is approximately 1 millimeter.

$= 9.9 \times 10^{-7}\ m$

2. If light of wavelength 6600 A has wave trains 20 long, what are its coherence length and coherence time?

Solution: Given: $\lambda = 6600 Å$, $N = 20 wave\ trains$, $Lcoh = ?, \tau_{coh} = ?$,

N is the number of wave trains and λ is the wavelength

Formula: $Lcoh = N \times \lambda$, $Lcoh = c \times \tau_{coh}$

Now, we can calculate the coherence length (L):

$$Lcoh = N \times \lambda = 20 \times 6600 \times 10^{-10} m = 1.32 \times 10^{-5} m = 0.0132mm = 13.2\mu m$$

Next, to find the coherence time(τ_{coh}) we use the formula:

$$Lcoh = c \times \tau_{coh}$$

$$\tau_{coh} = \frac{Lcoh}{c} = \frac{1.32 \times 10^{-5}}{3 \times 10^{8}} = 4.4 \times 10^{14} sec$$

So, the coherence length for the light is approximately 0.0132 millimeters, and coherence time is approximately 4.4 × 10^{14} sec

wavelength 10^{-9}

3. Compute the coherence length of light with 6328 A in 10-9sec pulse duration.

calulate the no. of wave trains

Solution: Given: $\lambda = 6328\text{Å}, \tau_{coh} = 10^{-9}sec$

N is the number of wave trains and λ is the wavelength

Formula $Lcoh = c \times \tau_{coh} = N \times \lambda$

To find the coherence length, we need to first calculate the coherence time (τ_{coh}) using the given pulse duration

Coherence length, $Lcoh = c \times \tau_{coh}$

$$Lcoh = 3 \times 10^8 \times 10^{-9} = 0.3m$$

Ans.

Now, we can also use the wavelength to calculate the coherence length

$\lambda = 6328 \text{ Å} = 6328 \times 10^{-10} \text{ m} = 6.328 \times 10^{-7} \text{ m}$

The coherence length can also be calculated as

$$Lcoh = c \times \tau_{coh} = N \times \lambda$$

$Lcoh = N \times \lambda$

$N = \frac{Lcoh}{\lambda} = 0.3$

Rearranging to solve for N, we get,

$$Lcoh = N \times \lambda$$

$$N = \frac{Lcoh}{\lambda} = \frac{0.3m}{6.328 \times 10^{-7}} = 474 = 4.7 \times 10^5$$

So, the coherence length for the light is approximately 0.3 meters, which corresponds to about 474 wavelengths.

4) A He-Ne laser, giving light at 6330 A has a coherence length of 20 km. Determine its coherence time and the number of waves per wave train.

Solution: Given: $Lcoh = 15\ km = 15000m\ \lambda = 6332\text{Å}, \tau_{coh} = ?$

N is the number of wave trains and λ is the wavelength

Formula $Lcoh = c \times \tau_{coh} = N \times \lambda$

$Lcoh = C \times \tau_{coh} = N \times \lambda = \frac{\lambda^2}{\Delta\lambda}$

First, let's calculate the coherence time (τ) using the formula:

$$Lcoh = c \times \tau_{coh}$$

$$\tau_{coh} = \frac{Lcoh}{c} = \frac{15000m}{3 \times 10^8 m/s} = 5 \times 10^{-5} sec$$

Next, to find the number of waves per wave train (N), we use the formula:

$$Lcoh = c \times \tau_{coh} = N \times \lambda$$

$$Lcoh = N \times \lambda$$

$$N = \frac{Lcoh}{\lambda} = \frac{15000}{6.332 \times 10^{-7}} = 2.3 \times 10^{10} = 23\ million$$

So, the coherence time for the He-Ne laser is approximately 5 x 10^{-5} seconds, and the number of waves per wave train is approximately 23 million.

5. A ruby laser emits light of wavelength 695.5 nm. If a laser pulse is emitted for 1.5 ×10-11sec and energy released per pulse is 0.15J (i) what is the length of the pulse? And (ii) how many photons are there in each pulse?

Solution: Given: $\lambda = 695.5\ nm, \tau_{coh} = 1.5 \times 10^{-11} sec, Lcoh = ?$

E = energy released per pulse is 0.15J,

N is the number of wave trains and λ is the wavelength

Formula $Lcoh = c \times \tau_{coh} = N \times \lambda$

i) Let's calculate length of the pulse,

$$Lcoh = c \times \tau_{coh}$$

$$Lcoh = 3 \times 10^{8} \times 1.5 \times 10^{-11} = 4.5 \times 10^{-3} m = 4.5mm$$

ii) Now, N is the number of wave trains and λ is the wavelength

First, calculate the energy of a single photon (E)

$$E = \frac{hc}{\lambda} = \frac{6.626 \times 10^{-34} \times 3 \times 10^{8}}{6.695 \times 10^{-9}} J = 2.83 \times 10^{-19} J$$

Next, calculate the number of photons

$$N = \frac{Energy\ released\ per\ pulse}{Energy\ of\ one\ photon} = \frac{0.15}{2.83 \times 10^{-19}} = 5.29 \times 10^{17} photons$$

So, there are approximately 5.29 x 10^{17} photons in each pulse.

6. The wavelength of He-Ne laser is 6328 nm. Its output power is 3.147mW. How many photons are emitted per minute when it is in operation?

Solution:

Given: $\lambda = 632.8\ nm = 632.8 \times 10^{-9}\ m, p = 3.147mW = 3.147 \times 10^{-3}, t = 1minute = 60sec\ \ n = ?$

Formula $Energy\ of\ a\ phton, E = \frac{hc}{\lambda}, h - planks\ constant$

If n is the number of photos, Energy of n number of photons is,

$$Energy\ of\ a\ phton, E = \frac{nhc}{\lambda} = p \times t$$

$$\frac{nhc}{\lambda} = p \times t$$

$$n = \frac{p \times \lambda \times t}{hc} = \frac{3.147 \times 10^{-3} \times 632.8 \times 10^{-9} \times 60}{6.63 \times 10^{-34} \times 3 \times 10^{8}}$$

$$n = 6 \times 10^{17}$$

6×10^{18}

7. Find the ratio of population of two energy levels in a laser if the transition between them produces light of wavelength 694.3 nm. Assume the ambient temperature to be ~~270 C~~. $27^{0}C$

Solution:

Given: $\lambda = 694.3nm = 694.3 \times 10^{-9}\, m, T = 27^{0}C = 300K,\ k = 1.38 \times 10^{-23}\, J/K,$
$h = 6.63 \times 10^{-34} Js$

Formula $\frac{N_2}{N_1} = e^{(E_2 - E_1)/kT} = e^{-\frac{hc}{\lambda kT}}$

$$\frac{N_2}{N_1} = e^{-\frac{6.63 \times 10^{-34} \times 3 \times 10^{8}}{694.8 \times 10^{-9} \times 1.38 \times 10^{-23} \times 300}}$$

$$\frac{N_2}{N_1} = 8.874 \times 10^{-31}$$

or bandwidth

8 Laser source with a wavelength of 488 nanometers (nm). The spectral width of the laser is measured to be 0.5 nm. What is the coherence length of this laser source?

Solution: Given: $\lambda = 488\ nm, \Delta\lambda$ *(spectral width)* = *0.5* nm $Lcoh = ?,$

Formula: . $Lcoh = c \times \tau_{coh} = \frac{\lambda^2}{\Delta\lambda}$ $= N \times \lambda$

$$Lcoh = \frac{\lambda^2}{\Delta\lambda}$$

$$Lcoh = \frac{\lambda^2}{\Delta\lambda} = \frac{488^2 nm}{0.5nm}$$

$$Lcoh = 238{,}144 \text{ nm or } 0.238 \text{ mm}$$

Therefore, the coherence length of the laser source is approximately 0.238 millimeters.

SHORT ANSWER TYPE QUESTIONS

1. Explain spontaneous and stimulated emission of radiation.
2. What is the importance of metastable state and pumping in the production of Laser?
3. Explain the following terms:
   (i) Induced Absorption and (ii) Population inversion.
4. What is stimulated emission? What role does it play in the operation of a laser?
5. What is pumping in LASER? Give the different types of pumping?
6. Why two level pumping is not suitable for obtaining population inversion?

7. What is active material in He-Ne laser? How population inversion is achieved in a He-Ne laser?
8. What is the role of resonant cavity in the operation of a LASER?
9. Explain the role of Helium in the He-Ne Laser.
10. Draw the energy level diagram of He-Ne laser. What is its wavelength in visible range?
11. Give the various uses of Lasers in medical, engineering and scientific field.
12. Explain the significance of coherence length in laser applications.
13. How does the spectral width of a laser affect its coherence length?
14. Compare the coherence lengths of a monochromatic and a broad-spectrum laser source.
15. Define coherence time and explain its relationship with laser pulse duration.
16. Explain the difference between coherence time and pulse duration in a laser.
17. What is the effect of increasing the wavelength of a laser on its coherence length?
18. Explain how the coherence time of a laser is related to its spectral bandwidth.
19. What is the effect of increasing the pulse duration of a laser on its coherence time?
20. How does the coherence time of a laser affect its ability to be used in optical communication systems?
21. What is LiDAR technology, and how does it work?
22. What are two common applications of LiDAR technology, and how is it used in each?
23. What is the advantage of using LiDAR over traditional surveying methods like GPS and leveling?
24. What is the primary function of a LiDAR sensor?
25. How does LiDAR measure distance to a target object?

Descriptive Answer Type Questions

1. Derive the formula for coherence length (Lcoh) in terms of wavelength (λ) and spectral width ($\Delta\lambda$), and explain the physical significance of each term. (5 marks)
2. What does the word LASER stand for? Explain main three processes involved in the production of LASER with appropriate figures.
3. With neat energy level diagram describe the construction and working of He-Ne Laser. What are its merits and demerits?
4. Write a note on Barcode reader
5. Describe the principles of LiDAR technology, including the use of laser pulses to measure distance
6. Compare and contrast 1D and 2D barcodes, including their applications, advantages, and limitations.
7. Write a note on LiDAR.

CHAPTER **2.**

Fibre Optics

2.1 INTRODUCTION TO FIBRE OPTICS

The communication of message or information to far off places in its fullest extent requires a reliable communication channel. The channel or waveguide refers either to a physical transmission medium such as a wire or to a logical connection medium such as a radio channel. The information is carried through the channel by a signal. The invention of telegraph by Samuel Morse in 1835 was the beginning of the era of electrical communications.

Alexander Graham Bell in 1880 invented a more scientific light communicating device known as photo phone which allowed transmission of speech on a beam of light. In 1895, Marconi demonstrated radio communication without using wires. To transmit information such as speech, images or data over a long distance a carrier wave is necessary. The information to be transmitted modulates the carrier wave. Though radio waves and microwaves which also are the forms of electromagnetic radiation are used as carrier waves to transmit signals over a long distance, coherent optical beams have more information carrying capacity because optical beams have frequencies in the range of 10^{14} to 10^{15} Hz.

The communication using light as signal carrier and optical Fibres as a medium of transmission is termed **optical Fibre communication**. The optical Fibre is a reliable light guide that works on the principle of total internal reflection and preserves the information transmitted on a beam of light through it.

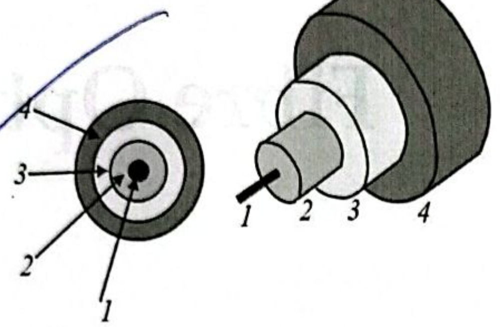
2.2 STRUCTURE OF OPTICAL FIBRES

An **optical Fibre** is a flexible, transparent glass or plastic Fibre slightly thicker than a human hair and it has a huge band width. It functions as a waveguide to transmit light between its launching and receiving ends.

A Fibre optic cable consists of a core, cladding, buffer and jacket. The **core** is a transparent inner most element of highest optical density that carries light. Its diameter is of the order of 8 μm to 100 μm. The core of the Fibre is surrounded by a transparent middle shell of slightly lower optical density is known as **cladding**. The lower refractive index of the cladding helps reflect light back to the core so that light retains within the core. The diameter of cladding is of the order of 125 μm.

The core and cladding of the Fibre is surrounded by several layers of **buffer coatings** of plastic or polymer. The buffer acts as shock absorber to protect core and cladding from damage. The diameter of the buffer coating is of the order of 250 μm.

The **outer jacket** around the buffer layers provides tensile strength to the cable to preven[t] pulling damage during installation. It also provides protection against moisture, abrasion, contamination and other environmental dangers. The diameter of the jacket is about 400 μm or 900 μm. The typical structure of a optical Fibre is shown in Fig. 2.1.



1. Core: 9 μm diameter
2. Cladding: 125 μm diameter.
3. Buffer: 250 μm diameter.
4. Jacket: 400 μm diameter

Fig: 2.1 Structure of a typical single-mode Fibre

2.3 Principle of Total Internal Reflection

The mechanism of light propagation along Fibre can be understood using the principle of geometrical optics.

Let n_1 be refractive index of optically denser medium and n_2 the refractive index of optically rarer medium.

Let a ray of light travelling from a optically denser to rarer medium with an angle of incidence i, then the refracted ray bends away from the normal in the rarer medium as shown in Fig. 2.2 (a).

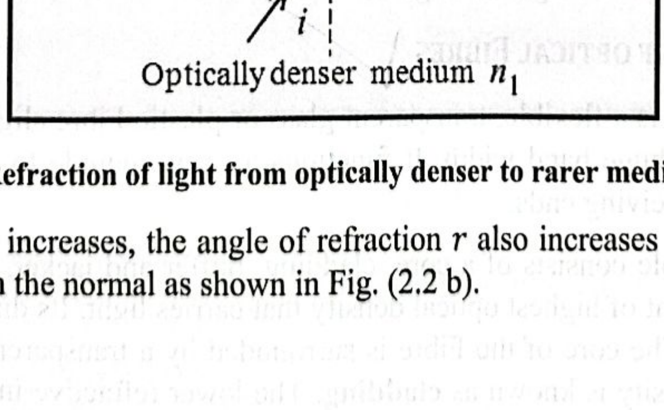


Fig. 2.2 (a) Refraction of light from optically denser to rarer medium

As the angle of incident i increases, the angle of refraction r also increases and the refracted ray bends more away from the normal as shown in Fig. (2.2 b).

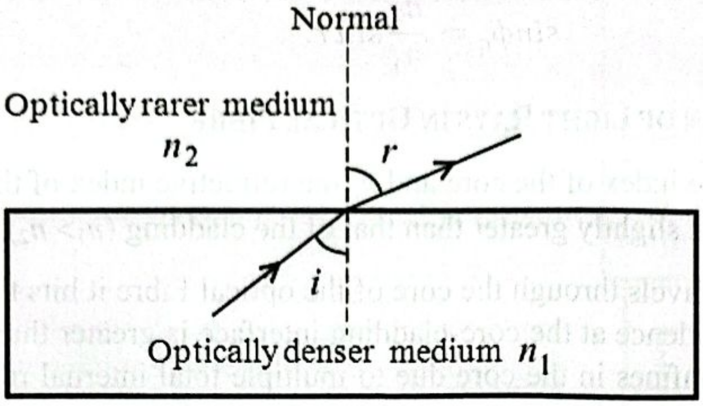


Fig. 2.2 (b)

In the latter case, at the particular angle of incidence the angle of refraction becomes equal to 90° which is known as ***critical angle*** ϕ_c as shown in Fig. (2.2 c).

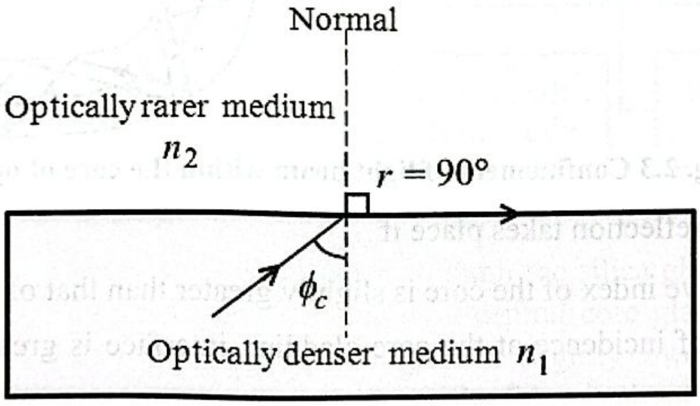


Fig. 2.2 (c) Angle of refraction $\angle r = 90°$ at $\angle i = \phi_c$

When the angle of incidence of a light ray is greater than the critical angle (i = ϕ_c) then the light rays bound in the same medium (optically denser medium).

Thus the beam of light confines in the same medium and undergoes multiple total internal reflections as shown in Fig. (2.2 d).

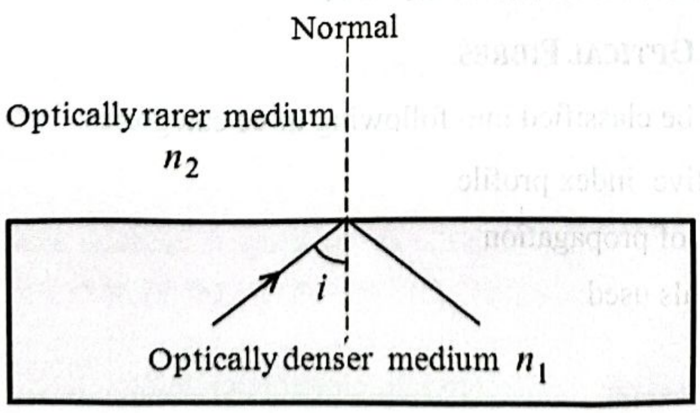


Fig. 2.2(d) Total internal reflection at $\angle i > \Phi_C$

Therefore the critical angle can be calculated using Snell's law,

$$n_1 sin\phi_c = n_2 \, sin \, r$$

$$\sin\phi_c = \frac{n_2}{n_1}\sin r$$

2.4 Propagation of Light Rays in Optical Fibre

Let n_1 be refractive index of the core and n_2 the refractive index of the cladding. The refractive index of the core is slightly greater than that of the cladding ($n_1 > n_2$).

As a ray of light travels through the core of the optical Fibre it hits the core-cladding interface. If the angle of incidence at the core-cladding interface is greater than that of the critical angle, the ray of light confines in the core due to multiple total internal reflections as shown in Fig. (2.3).

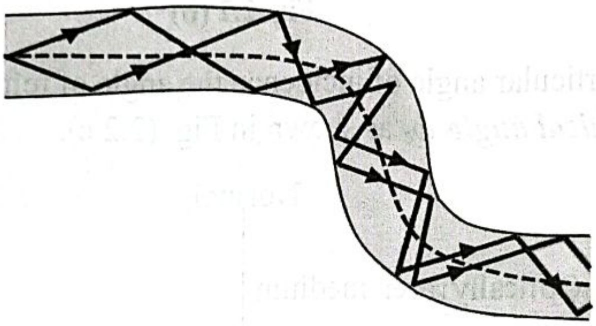


Fig. 2.3 Confinement of light beam within the core of optical Fibre

The total internal reflection takes place if

(a) The refractive index of the core is slightly greater than that of the cladding.

(b) The angle of incidence at the core-cladding interface is greater than the critical angle (ϕ_c).

∴ According to Snell's law, $n_1 \sin\phi_c = n_2 \sin 90^o$

$$\sin\phi_c = \frac{n_2}{n_1} \qquad (\because \sin 90^o = 1)$$

Thus the light rays whose angle of incidence at the interface of core and cladding is greater than the critical angle travel forward through the Fibre by multiple total internal reflections and emerge out from the receiver end of the Fibre.

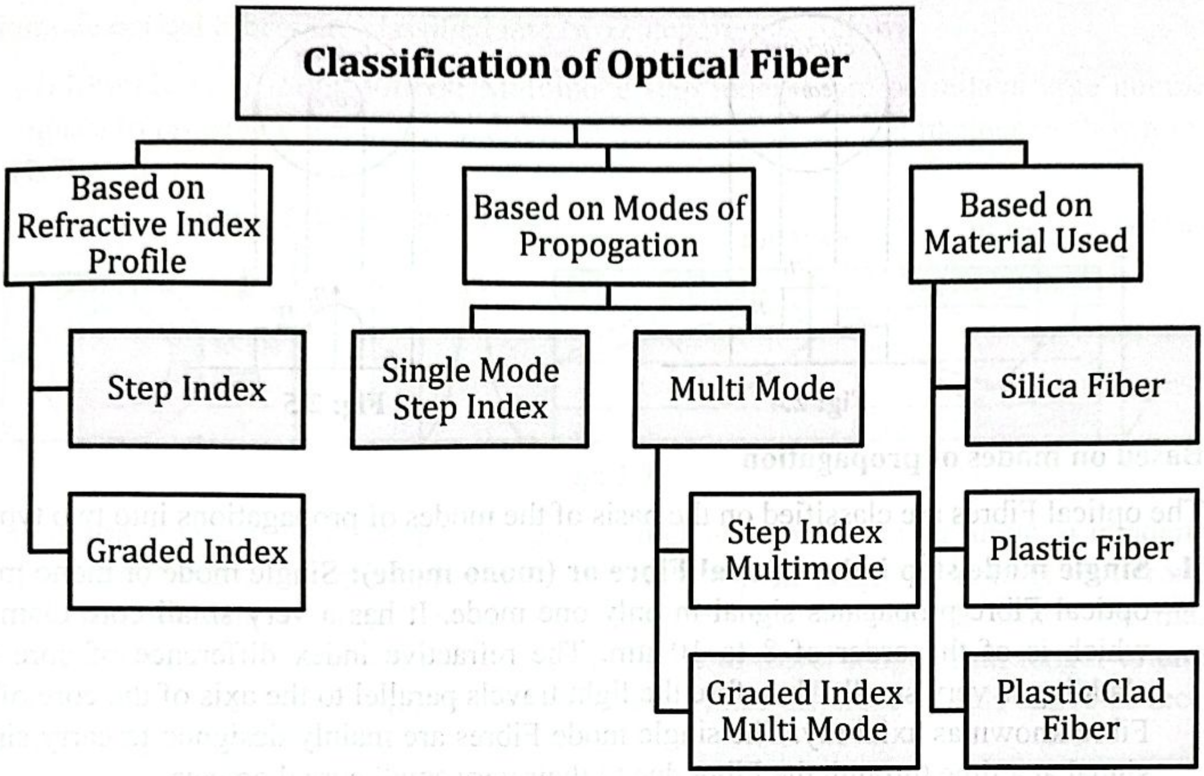
2.5 Classifications Optical Fibres

The optical Fibres can be classified into following three categories.

(a) Based on refractive index profile

(b) Based on modes of propagation

(c) Based on materials used



(a) Based on refractive index profile

The light or the optical signals can be propagated through the silica glass Fibres by total internal reflection. A typical glass Fibre consists of a central core glass of the order of diameter 50 μm surrounded by the cladding of a glass of slightly lower refractive index than the core refractive index. The overall diameter of the Fibre is about 125 to 250 μm.

On the basis of the refractive index profile optical Fibres are classified into two categories.

1. **Step index Fibre:** In the step index Fibre the refractive index of core medium is uniform throughout it and changes abruptly at the core-cladding boundary. The light rays propagating through the Fibre are either in the form of meridional rays or axial rays. The meridional rays cross the Fibre axis for every reflection at the core cladding boundary and propagate in a zig-zag manner as shown in figure (2.4).
2. **Graded index Fibres:** In the graded index Fibre the refraction index of the core is not constant but decreases gradually from its maximum value at the centre of the core. The decrease in refractive index with radius minimizes dispersion of light. The ray entering the core undergoes refraction from an optically denser layer to rarer layer and in this process it goes on bending at every refraction within the core known as helical or skew rays as shown in figure (2.5).

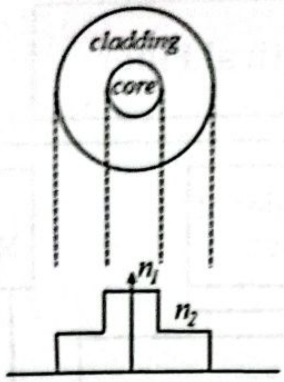


Fig: 2.4

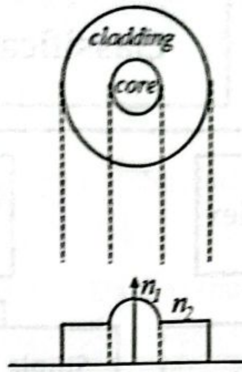


Fig: 2.5

(b) Based on modes of propagation

The optical Fibres are classified on the basis of the modes of propagations into two types

1. **Single mode step index optical Fibre or (mono mode):** Single mode or mono mode optical Fibre propagates signal in only one mode. It has a very small core diameter which is of the order of 8 to 10 μm. The refractive index difference of core and cladding is very small. Therefore the light travels parallel to the axis of the core of the Fibre known as axial ray. The single mode Fibres are mainly designed to carry single signal at a time through the Fibre due to their very small core diameter.

   The fabrication of single mode Fibres is very difficult so these types of Fibres are costly. Further the launching of light into single mode Fibres is also difficult due to their shorter diameter. In this mode optical Fibre suffers little pulse dispersion known as *intramodal dispersion*.

   Generally in the single mode Fibres the transmission loss of the signal is very small. The input and output pulse propagating through the Fibre is as shown in figure (Fig. 2.6).

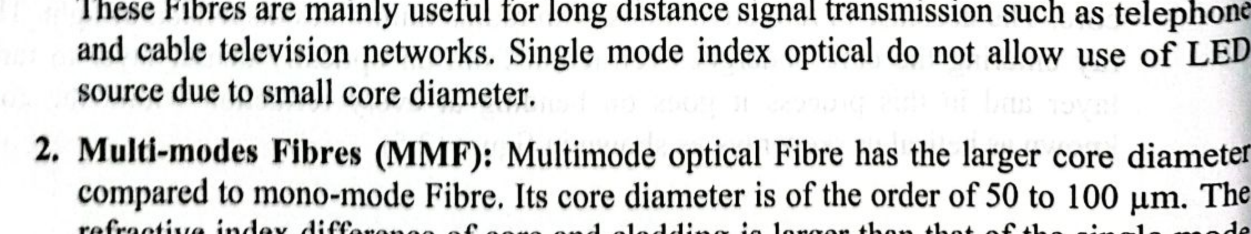


Fig: 2.6

   These Fibres are mainly useful for long distance signal transmission such as telephone and cable television networks. Single mode index optical do not allow use of LED source due to small core diameter.

2. **Multi-modes Fibres (MMF):** Multimode optical Fibre has the larger core diameter compared to mono-mode Fibre. Its core diameter is of the order of 50 to 100 μm. The refractive index difference of core and cladding is larger than that of the single mode Fibre. The multimode optical Fibres are mainly designed to carry multiple signals at a time through the Fibre due to their larger core diameter. In these Fibres the signals emerge out at receiver end are at different times. As a result the multimode optical Fibre creates larger pulse dispersion in compare with single mode Fibre. Multi-mode index optical Fibres permit use of light emitting diodes (LED) due to larger core diameter.

Multimode optical Fibres are classified into two categories as follows:

(i) Multimode step index Fibres: Multimode step index Fibre permits a large number of signals to propagate through it in different modes and in zig-zag manner as shown in Fig. (2.7).

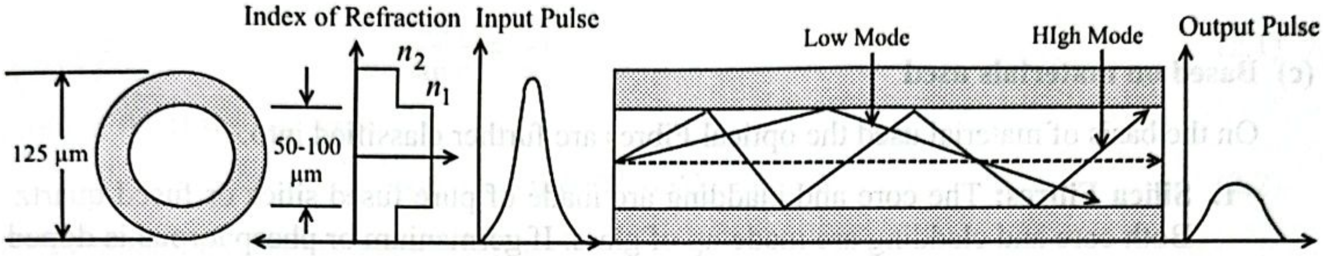


Fig: 2.7

In multimode step index Fibre the signals which are making larger angle of incidence at the core-cladding interface have to travel shorter path length to emerge out from the receiver end known as low mode. On the other hand the signals which are making smaller angle at the core-cladding interface have to travel longer path length to emerge out from the receiver end known as high mode. Thus different signals arrive at receiver end at slightly different times.

This effect is known as ***intermodal dispersion.*** Therefore the multimode step index Fibres suffer intermodal dispersion. The input and resultant output pulse propagating through the Fibre is as shown in Fig. (2.7). Thus these types of Fibres are best suited for transmission of signal over a shorter distance such as in an endoscope.

(ii) Multimode graded index optical Fibres: Multimode graded index Fibre allows a large number of signals to propagate through it. The angle of incidence of the signal at the region of higher refractive index region to the region of lower continuously bent and propagated in the form of skew rays or helical manner as shown in Fig. (2.8).

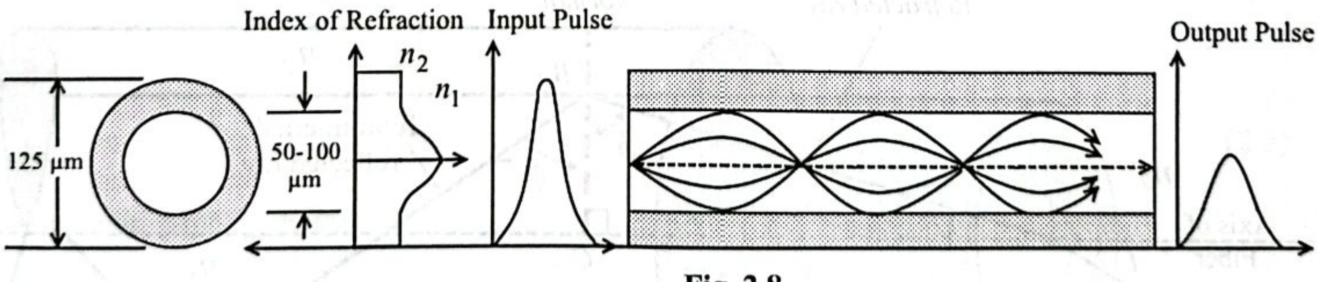


Fig. 2.8

In the multimode graded index optical Fibre the signals which are making larger angle of incidence at the region of higher refractive index to the region of lower refractive index have to travel larger path length and travel with a higher speed of propagation. On the other hand the signals which are making smaller angle of incidence have to travel shorter path length and travel with a smaller speed of propagation.

In this case of Fibre the signal distortion is very low due the self-focusing effect of the signal along the axis of the core. Here the light rays travel at different speeds and in different paths through the Fibre because of the parabolic variation of refractive index of the core. As a result the signals are continuously refocused and all the signals reach at receiver end of the Fibre at the same time.

Therefore multimode graded index optical Fibres suffer less intermodal dispersion as compare to multimode step index optical Fibres. The input and the resultant output pulse propagating through the Fibre are shown in Fig. (2.8). These Fibres are therefore most suitable for transmission of the signal over a shorter distance such as local area networks.

(c) Based on materials used

On the basis of material used the optical Fibres are further classified into:

1. **Silica Fibres:** The core and cladding are made of pure fused silica or fused quartz. Both core and cladding are made up of glass. If germanium or phosphorous is doped in silica, its refractive index increases. Silica is doped with boron or fluorine to decrease the refractive index. Glass Fibres are preferred due to their low intrinsic absorption at wavelengths of operation. Addition of any other impurity can cause attenuation and scattering of light propagating through the Fibre.
2. **Plastic Fibres:** In this case both the core and the cladding are made up of plastic. The core is of polystyrene and methyl methacrylate is used as cladding. Plastic Fibres are light and flexible. They are widely used in short distance applications.
3. **Plastic-clad Fibres:** The plastic clad silica Fibres are composed of glass core and polymer as cladding. They are lighter, flexible, and cheap. However in PCS Fibres losses are more. PCS Fibres are suitable for shorter distances.

2.6 Acceptance Cone and Numerical Aperture of a Fibre

Lets us consider a step index optical Fibre with light propagating as shown in Fig. (2.9).

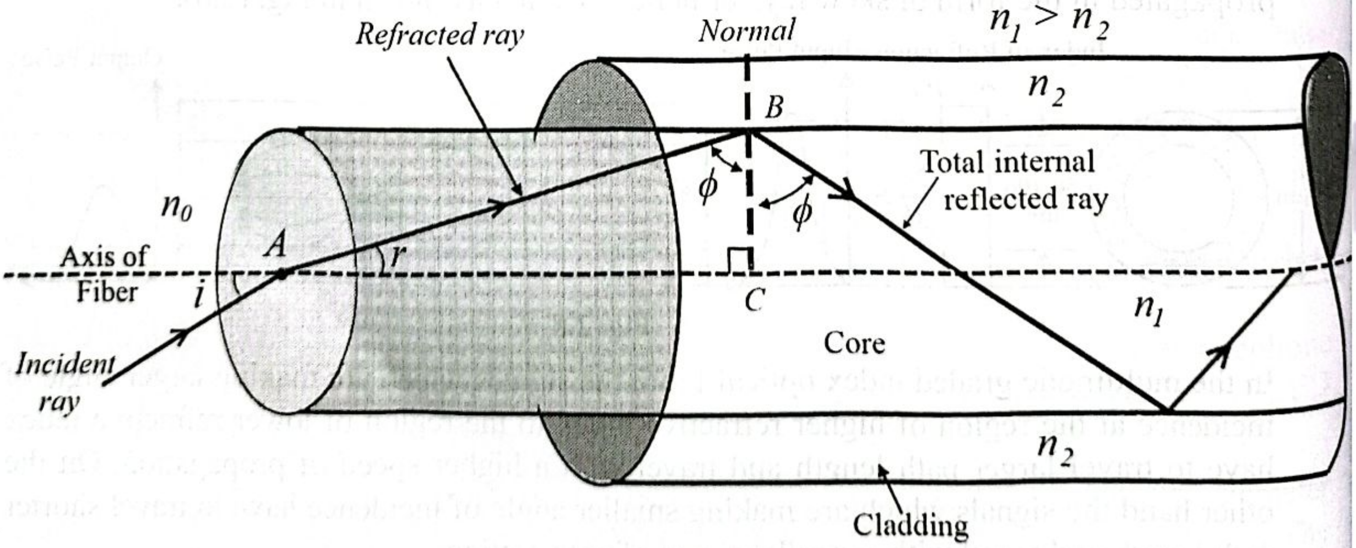


Fig: 2.9

Let n_1 be refractive index of the core, n_2 the refractive index of the cladding and n_0 be the refractive index of launching medium. The ray of light enters the core at an angle of incidence i to the axis of the core. It gets refracted at an angle of refraction r and incidence at the core-cladding interface at an angle ϕ to the normal.

If the angle of incidence at the core -cladding interface is greater than critical angle (ϕ) then the light ray bound in the core medium and undergoes total internal reflection. Since the refractive index of core is slightly greater than that of the cladding.

Applying Snell's law at launching face of the optical Fibre

$$\frac{\sin i}{\sin r} = \frac{n_1}{n_0} \tag{2.1}$$

From Δ ACB we can write,

$$\sin r = \sin(90^0 - \phi) = \cos\phi \tag{2.2}$$

Using equation (2.2) in equation (2.1),

$$\frac{\sin i}{\cos\phi} = \frac{n_1}{n_0}$$

$$\sin i = \frac{n_1}{n_0}\cos\phi \tag{2.3}$$

According to the principle of total internal reflection, let the angle of incidence at core-cladding interface $\phi = \phi_c$. So, the above equation (2.3) can be rewritten as

$$\frac{\sin i}{\cos\phi_c} = \frac{n_1}{n_0}$$

$$\sin i = \frac{n_1}{n_0}\cos\phi_c \tag{2.4}$$

Now applying Snell's law at interface of core and cladding we get, if $i = \phi_c$, then angle of refraction $(r) = 90^o$,

$$\frac{\sin i}{\sin r} = \frac{n_2}{n_1}$$

$$\therefore \frac{\sin\phi_c}{\sin 90} = \frac{n_2}{n_1}$$

$$\sin\phi_c = \frac{n_2}{n_1} \qquad (sin 90^0 = 1) \tag{2.5}$$

We have, $\sin^2\phi_c + \cos^2\phi_c = 1$

$$\therefore \cos\phi_c = \sqrt{1 - \sin^2\phi_c}$$

Put $sin\ \phi_c = \frac{n_2}{n_1}$ in above relation,

$$\cos\phi_c = \sqrt{1 - \frac{(n_2)^2}{(n_1)^2}}$$

$$\cos\phi_c = \frac{\sqrt{n_1{}^2 - n_2{}^2}}{n_1} \tag{2.6}$$

Putting equation (2.6) in equation (2.4)

$$\sin i = \frac{n_1}{n_0} \times \frac{\sqrt{{n_1}^2 - {n_2}^2}}{n_1}$$

$$\sin i = \frac{\sqrt{{n_1}^2 - {n_2}^2}}{n_0} \tag{2.7}$$

In this case the incident ray of light is launched from the air medium then put $n_0 = 1$ in the equation (2.7),

$$\sin i = \sqrt{{n_1}^2 - {n_2}^2} \tag{2.8}$$

Acceptance angle and acceptance cone

The **acceptance angle** is the maximum angle of incidence at or below which the light rays undergo total internal reflection. In three dimensions, the light rays enclosed within the cone and transmitted through the optical Fibre are known as ***acceptance cone.***

$$\sin i = \frac{\sqrt{{n_1}^2 - {n_2}^2}}{n_0} \tag{2.9}$$

where n_0 is the refractive index of launching medium of the light rays.

Numerical aperture (N.A.)

Numerical aperture of a Fibre is defined as the light gathering capacity of the optical Fibre. The amount of light accepted by the optical Fibre is proportional to acceptance angle.

Numerical aperture is equal to sine of acceptance angle.

i.e. Numerical aperture = sin (i) (2.10)

Put equation (2.9) in equation (2.10)

$$\text{N.A.} = \frac{\sqrt{n_1^2 - n_2^2}}{n_0} \tag{2.11}$$

The refractive index of air medium n_o is equal to unity

$$\text{N.A.} = \sqrt{n_1^2 - n_2^2} \tag{2.12}$$

Fractional change in refractive index (Δ)

It is the ratio of the difference of refractive indices of core and cladding to refractive index of the core.

$$\Delta = \frac{n_1 - n_2}{n_1} \tag{2.13}$$

We have

$$\text{N.A.} = \sqrt{n_1^2 - n_2^2}$$

$$\text{N.A.} = \sqrt{(n_1 + n_2)(n_1 - n_2)}$$

$$\text{N.A.} = \sqrt{\frac{(n_1 + n_2)}{2} \frac{(n_1 - n_2)}{n_1} \times 2n_1} \tag{2.14}$$

The refractive index core is slightly less than that of refractive index of cladding hence $n_1 \approx n_2$ $\left(\frac{n_1+n_2}{2} \approx n_1 \text{ and } \Delta = \frac{n_1-n_2}{n_1}\right)$

Equation (2.14) becomes,

$$N.A. = \sqrt{n_1 \times \Delta \times 2n_1}$$

$$N.A. = \sqrt{2n_1^2\Delta}$$

$$N.A. = n_1\sqrt{2\Delta} \tag{2.15}$$

The value of numerical aperture determines amount of light accepted by the optical Fibre and it lies between 0.13 and 0.50. It has been observed that Fibres whose numerical aperture value lies in range of 0.1 to 0.3 are used for long distance communication. On the other hand, Fibres whose numerical aperture values are in range of 0.4 to 0.5 are used for short distance communication.

2.7 V- NUMBER

An optical Fibre is characterized by one more parameter known as V-number or cut-off or normalized frequency of the Fibre.

V- number is given by the following relation

$$V = \frac{2\pi a}{\lambda_0} \times \sqrt{n_1^2 - n_2^2} \tag{2.16}$$

where, a is the radius of the core of the Fibre and

λ_0 is the free space wavelength of the light.

We have, Numerical aperture $= \sqrt{n_1^2 - n_2^2} = n_1\sqrt{2\Delta}$

Equation (2.16) can be written as

$$V = \frac{2\pi a}{\lambda_0} \times n_1\sqrt{2\Delta} \tag{2.17}$$

The maximum number of modes N_{max} allowed by the step Fibre is given by

$$N_{max} = \frac{1}{2}V^2 \tag{2.18}$$

In case the normalized frequency is less than 2.405 the Fibre allows only single mode.

2.8 CUT-OFF WAVELENGTH

Cut-off wavelength is the shortest wavelength at which only fundamental (single) mode can propagate in a multimode optical Fibre. It can be calculated by putting $V = 2.405$ and $\lambda_0 = \lambda_c$ in the equation (2.36).

$$2.405 = \frac{2\pi a}{\lambda_c} \times \sqrt{n_1^2 - n_2^2}$$

$$\lambda_c = \frac{2\pi a}{2.405} \times \sqrt{n_1^2 - n_2^2} \tag{2.19}$$

In case of ***graded index Fibre*** V is greater than 2.405.

The maximum numbers of modes allowed by the graded index Fibre is given by,

$$N_{max} = \frac{1}{4}V^2 \tag{2.20}$$

2.9 DIFFERENCE BETWEEN SINGLE MODE AND MULTI-MODE OPTICAL FIBRE

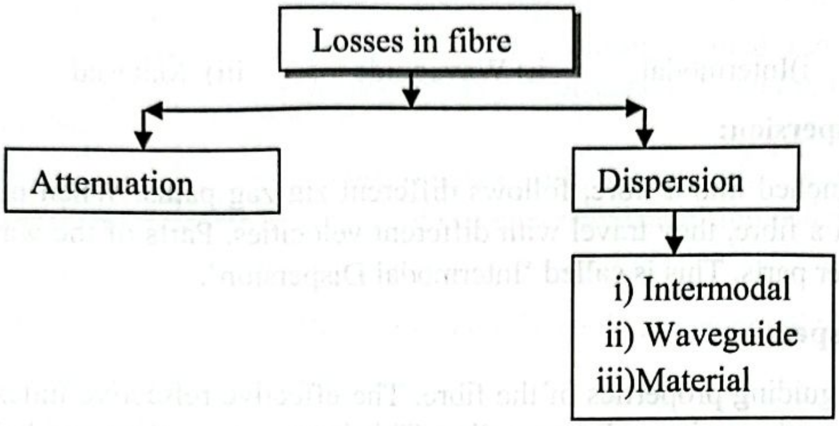
Single-mode optical Fibre	Multimode optical Fibre
Diameter of the core is very narrow of the order of ~ 10μm.	Diameter of the core is in the range of 50 to 100 μm.
Only one path for propagation of ray is allowed.	Large number of paths is allowed for propagation of ray.
Difference between the refractive index of the core and cladding is very small.	Difference between the refractive index of the core and cladding is large.
There is very less attenuation and zero intermodal dispersion of the signal.	There is attenuation and comparatively larger dispersion of the signal.
Bandwidth of the optical Fibre is very high about greater than 3GHz-km.	Bandwidth of the optical Fibre is less than 200 MHz-km.
Used for long distance communication.	Used for short distance communication.
Its installation is costlier.	It is comparatively cheaper.

2.10 DIFFERENCE BETWEEN STEP-INDEX AND GRADED-INDEX OPTICAL FIBRE

Step-Index optical Fibre	Graded-Index optical Fibre
Refractive index of the core is uniform and abruptly changes at the core-cladding interface.	Refractive index of the core is non-uniform and it decreases parabolically from the centre of axis to the core-cladding interface.
Rays propagate in form of axial or meridional rays inside the core which crosses the Fibre axis at every reflection.	Rays propagate in form of helical or skew rays which do not cross the Fibre axis.
Normalized frequency or V-number is less than 2.405.	Normalized frequency or V-number is greater than 2.405.
Number of allowed modes, $N_{max} = \frac{1}{2}V^2$	Number of allowed modes, $N_{max} = \frac{1}{4}V^2$
Attenuation and dispersion is high for multimode step-index Fibre.	Attenuation and dispersion is comparatively lower for graded-index Fibre.

2.11 LOSSES IN FIBRE

There are mainly 2 types of losses as per the following chart given below:



2.11.1 ATTENUATION

Attenuation is the reduction in intensity or amplitude of a signal as it travels through a medium, such as a Fibre optic cable or air. This phenomenon is a natural consequence of signal transmission and can have significant impacts on communication systems. Attenuation can be measured in decibels (dB) and is a critical factor in determining the performance and reliability of signal transmission.

Causes of Fibre Attenuation:

1. Absorption: Signal energy is absorbed by the Fibre material, converting it into heat.

2. Scattering: Signal energy is scattered in different directions, reducing the intensity of the signal.

3. Reflection: Signal energy is reflected back to the source, causing signal loss.As the ray propagates through Fibre, it gets attenuated.

Mathematically, attenuation can be expresses as:

$$\alpha = \frac{10}{L}.\log\left(\frac{P_{in}}{P_{out}}\right) \text{ dB/km} \tag{2.21}$$

Attenuation constant (α): It is defined as the ratio of optical output power from a Fibre of length L to input optical power.

In case of ideal Fibre input power is equal to output power.

Factors Affecting Fibre Attenuation:

1. Fibre Material: Different materials have varying levels of absorption and scattering.

2. Wavelength: Attenuation varies with wavelength, with shorter wavelengths experiencing higher attenuation.

3. Fibre Length: Longer Fibres experience higher attenuation due to increased absorption and scattering.

4 Dispersion: A light pulse launched into an optical Fibre decreases in amplitude due to losses in the Fibre. It also spreads during its travel. The output pulse is wider than the input pulse. This process is called 'dispersion' & it is measured in ns/km. There are three types of dispersion.

i)Intermodal ii) Waveguide iii) Material

i) Intermodal Dispersion:

A ray of light launched into a fibre, follows different zig-zag paths. When numerous modes are propagation in a fibre, they travel with different velocities. Parts of the wave arrive at the output, before other parts. This is called 'Intermodal Dispersion'.

ii) Waveguide Dispersion:

It arises from the guiding properties of the fibre. The effective refractive index for any mode varies with wavelength causing pulse spreading. This is known as 'Waveguide Dispersion'.

iii)Material Dispersion:

Light waves of different wavelength travel at different speeds in a medium. The short wavelength wave travel slower than long wavelength waves therefore narrow pulses of light tend to broaden as they travel in the optical fibre. This is known as 'Material Dispersion'. Clearly, the spectral width of the source determines the extent of material dispersion.

2.12 ADVANTAGES OF OPTICAL FIBRE COMMUNICATION

The Fibre optic system has wide applications in the telecommunication industry to rapidly develop new advancements in technology. It offers many advantages over the traditional metal (copper) wire as listed below.

1. Optical Fibres are made of silica (SiO_2) which is abundantly available on the earth.
2. The optical Fibres require low power transmitters as against the high-voltage transmitters needed for copper wires. Example: LED and Semiconductor LASER diode.
3. Message is transmitted in the form of light. Therefore there is no danger of electrical fire as with copper wires.
4. They can be bent or twisted without damage as compared to the copper wires.
5. Optical Fibres are light-weight, flexible and compact than metal wires. Therefore very less space is required.
6. The Fibres are made up of dielectric material. Therefore they are electrically insulator and do not require isolation coating.
7. Optical Fibre cables are unaffected by interference originating from power cables, radio waves, railway power lines etc.
8. There is no effect of moisture corrosion or nuclear radiation on the optical Fibres.
9. Optical Fibre cables can be operated over high temperature ranges.
10. The optical Fibre communication system is more secure because there is no cross talk and therefore privacy of information is maintained.
11. The high bandwidth of light provides the optical Fibre with a very large amount of information carrying capacity.

12. In the modern optical Fibre telecommunication systems, the Fibres used have a transmission loss of only 0.002 dB/km.

2.13 Disadvantages of Optical Fibre Communication

1. Despite the fact that the raw material is very cheap, optical Fibres are still more expensive than copper wires because costly repeaters are required in long distance communication.
2. Optical Fibres cannot be connected together (spliced) as easily as copper cables and requires additional training of personnel and expensive precision splicing.
3. Expensive special test equipments are often required.
4. Optical Fibres require more protection around the cable as compared to the copper wires.

2.14 Applications of Optical Fibres

The information carrying capacity of optical Fibre is very high and communication is very fast even for longer distances. The optical Fibres are thin, light weight, requires very less space, and signal loss is very less compared to copper wire communication system.

Following are some of the applications.

(a) **Military application:** The compact, easy to carry light weight, high transmission but low loss equipments of optimum performance used in the battlefield conditions can be fabricated using Fibre optic cables for military applications.

(b) **Medicine:** The doctors use **endoscope** to examine internal body organ of the patient. To examine inside of throat and lungs doctors use **bronchoscope** so that growth of any tumor can be detected. **Laparoscope** helps doctors to examine abdomen and pelvic area of human body. All these methods are painless and provide the doctors correct information to evaluate internal body parts.

(c) **Inspection of machine parts:** The engines of road vehicles as well as aeroplanes, rockets, space shuttles can be inspected for wear and tear using Fibre optics as part of imaging systems.

(d) **Semiconductor equipment:** Fibre optics are used as interconnects in semiconductor equipments and sensors used in a variety of applications.

(e) **Telemetry:** Monitoring the operations of electronically operated machinery over short or long distances by remote control is known as telemetry. Optical Fibres are used in telemetry.

(f) **Broadcasting:** Fibre optics are designed to offer multiple channel and high-bandwidth links as well as power connections to and from cameras, trucks, and satellite links which improves the broadcasting to a greater extent.

(g) **Closed circuit television:** CCTV cameras employed in various applications make use of optical Fibres.

(h) **Fibre optics for Education:** In the field of education Fibre optic technology is used in the research institutes, universities, colleges and schools to set up network that connects students and teachers.

2.15 Communication Through Optical Fibre

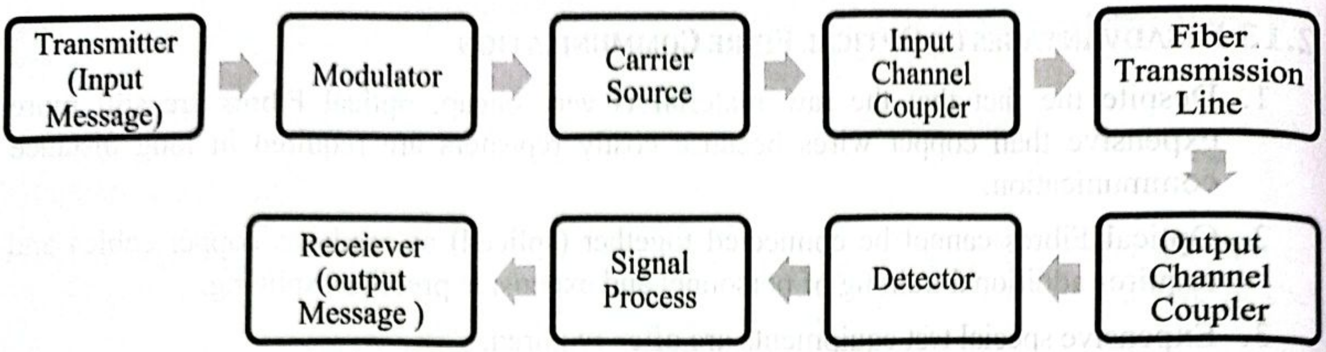


Fig. 2.10

Fibre optic communication refers to the medium and the technology associated with the transmission of information as light pulses along a Fibre, approximately 50-100 micrometer in diameter. Laser light in the form of pulses or flashes carries the signals along the optical Fibre. The laser light source is turned ON and OFF very quickly. The light pulses make up a binary code which is decoded by the receiver.

Optical Transmitter

The input is an electrical signal which is converted into either an electronic or an optical signal.

The optical transmitter comprise of following parts:

(a) **Modulator:** The modulator converts the electrical message into a proper format and then that signal is modulated onto the wave generated by the carrier source.

(b) **Carrier Source:** The carrier source generates the wave called the carrier wave through which the information is transmitted. Semiconductor laser diode or light-emitting diode (LED) can be used as carrier source.

(c) **Input Channel Coupler:** The function of the coupler is to feed power into the communication channel. The Fibre optic communication system requires that the coupler must efficiently transfer the modulated light beam from the source to the optic Fibre. However this transfer is accomplished with large reduction in power. To overcome loss of data couplers are preferred.

Communication Channel

The communication channel transports the optical signal from the transmitter to the receiver without distorting it. In order to transmit light with a relatively small amount of power loss optical Fibre is established as a good communication channel.

Optical Receiver: The optical receiver consists of

(a) **Output Channel Coupler:** The output coupler directs the light emerging from the Fibre onto the light detector. This light is radiated in a pattern identical to the Fibre's acceptance cone.

(b) **Detector:** The data/information being transmitted is now separated from the carrier wave. A photo detector in the Fibre system converts the light wave into an electric current.

(c) **Signal Processor:** In analogue transmission, the signal processor carries amplification and filtering of the signal. Also it is necessary to block any other undesired frequencies from further travel.

SOLVED PROBLEMS

Example 1: Calculate the numerical aperture of a Fibre with core index (n_1) =1.61 and cladding index (n_2) =1.55.

Solution: **Given**

Refractive index of core (n_1) =1.61

Refractive index of cladding (n_2) =1.55

Formula: Numerical Aperture $N.A = \sqrt{n_1{}^2 - n_2{}^2}$

$$= \sqrt{(1.61)^2 - (1.55)^2}$$

Numerical aperture = 0.43

Example 2: Compute the numerical aperture, acceptance angle and the critical angle of the Fibre having refractive indices of core and cladding is 1.5 and 1.45 respectively.

Solution: **Given**

Refractive index of core (n_1) =1.5

Refractive index of cladding (n_2) =1.45

Formula: Calculation of Numerical aperture

$$N.A = \sqrt{n_1{}^2 - n_2{}^2}$$

$$= \sqrt{(1.5)^2 - (1.45)^2}$$

Numerical aperture = 0.38

Calculation of acceptance angle

$\sin i$ = Numerical aperture

$$\sin i = 0.38$$

$$i = sin^{-1}(0.38)$$

Acceptance angle (i) = 22.58°

Calculation of critical angle (ϕ_c)

$$sin\phi_c = \frac{n_2}{n_1}$$

$$sin\phi_c = \left(\frac{1.45}{1.5}\right)$$

$$\phi_c = sin^{-1}\left(\frac{1.45}{1.5}\right)$$

Critical angle (ϕ_c) = 75. 16°

Example 3: Internal critical angle in core cladding interface of a step index optical Fibre is 80.6°. Calculate maximum acceptance angle if refractive index of cladding is 1.48.

Solution: **Given**

Critical angle (ϕ_c) = 80.6°

Refractive index of cladding (n_2) =1.48

Formula: According to Snell's law,

$$\sin \phi_c = \frac{n_2}{n_1}$$

$$\sin (80.6) = \frac{1.48}{n_1}$$

$$n_1 = \frac{1.48}{\text{Sin } (80.6)} = 1.50$$

$$sin(i)_{max} = \sqrt{{n_1}^2 - {n_2}^2}$$

$$sin(i)_{max} = \sqrt{(1.50)^2 - (1.48)^2} = 0.0596$$

$$(i)_{max} = \sin^{-1}(0.0596)$$

Maximum acceptance angle (i) $_{max}$= 3.41

Example 4: If refractive index of core of a Fibre optic cable is 1.5 and fractional index difference Δ is 0.0005, find refractive index of cladding and numerical aperture.

Solution: **Given**

Fractional refraction index difference (Δ) = 0.0005

Refractive index of core (n_1) =1.5

Formula: Fractional refraction index difference

$$\Delta = \frac{n_1 - n_2}{n_1} \Longrightarrow 0.0005 = \frac{1.5 - n_2}{1.5}$$

Refractive index of cladding (n_2) = 1.49

$$\text{Numerical aperture} = \sqrt{{n_1}^2 - {n_2}^2} = \sqrt{(1.5)^2 - (1.49)^2}$$

Numerical aperture = 0.17

Example 5: Determine the refractive index of cladding material of a Fibre. Given fractional change in refractive index is 12 × 10^{-3} and numerical aperture is 0.22.

Solution: **Given**

Fractional change in index (Δ) = 12 × 10^{-3}

Numerical aperture = 0.22

Formula: Calculation of refractive index of core (n_1)

$$\text{Numerical aperture} = \mu_1\sqrt{2\Delta}$$

$$0.22 = \mu_1\sqrt{2 \times 12 \times 10^{-3}}$$

Refractive index of core (n_1) = 1.42

Calculation of refractive index of cladding (n_2)

Fractional change in refractive index (Δ) = $\frac{n_1-n_2}{n_1}$

$$12 \times 10^{-3} = \frac{1.42 - n_2}{1.42}$$

Refractive index of cladding (n_2) = 1.40

Example 6: The numerical aperture of an optical Fibre is 0.5 and core reflective index is 1.54. Find refractive index of the cladding.

Solution: **Given**

Numerical aperture = 0.5

Core refractive index (n_1) = 1.54

Formula: Numerical Aperture (N.A.) = $\sqrt{n_1{}^2 - n_2{}^2}$

$$0.5 = \sqrt{(1.54)^2 - n_2{}^2}$$

$$(0.5)^2 = (1.54)^2 - n_2{}^2$$

Refractive index of cladding (n_2) = 1.45

Example 7: A step index Fibre has a core diameter of 29×10^{-6} m. the refractive indices of core and claddings are 1.52 and 1.5189 respectively. If the light of wavelength 1.3 μm is transmitted through the Fibre, determine normalized frequency of the Fibre.

Solution: **Given**

Diameter of core (d) = 29×10^{-6} m

Refractive index of core (n_1) = 1.52

Refractive index of cladding (n_2) = 1.5189

Wavelength (λ) = 1.3 μm = 1.3×10^{-6} m

Formula: The normalized frequency is given by,

$$V = \frac{\pi d}{\lambda}\sqrt{n_1{}^2 - n_2{}^2}$$

$$V = \frac{\pi \times 29 \times 10^{-6}}{1.3 \times 10^{-6}}\sqrt{(1.52)^2 - (1.5189)^2}$$

Normalized frequency of the Fibre (V) = 4.05 ≈ 4

Example 8: Calculate the numerical aperture and hence the acceptance angle for an optical Fibre. Given that the refractive indices of the core and the cladding are 1.45 and 1.40 respectively.

Solution: **Given**

Refractive index of the core (n_1) = 1.45

Refractive index of cladding (n_2) = 1.40

Formula: Numerical aperture $= \sqrt{n_1{}^2 - n_2{}^2}$

$$= \sqrt{(1.45)^2 - (1.40)^2}$$

Numerical aperture = 0.37

$\sin(i)$ = Numerical aperture

$$\sin(i) = \sqrt{n_1{}^2 - n_2{}^2} = 0.37$$

$$i = \sin^{-1}(0.37)$$

Acceptance angle (i) = 22.17$^{0'}$

Example 9: Compute the maximum radius allowed for a Fibre having core refractive index 1.47 and cladding refractive index 1.46. The Fibre supports only one mode at a wavelength of 1300 nm.

Solution:

Given

Refractive index of the core (n_1) = 1.47
Refractive index of cladding (n_2) =1.46
Wavelength (λ) = 13 nm =1300 × 10^{-9} m

Formula: The normalized frequency

$$(V) = \frac{2\pi a}{\lambda}\sqrt{n_1{}^2 - n_2{}^2}$$

If V ≤ 2.405 then optical Fibre can support single mode.

$$2.405 = \frac{2\pi a}{1300 \times 10^{-9}}\sqrt{(1.47)^2 - (1.46)^2}$$

$$a = \frac{2.405 \times 1300 \times 10^{-9}}{2\pi \times \sqrt{(1.47)^2 - (1.46)^2}}$$

Radius of the core (a) = 2.9 ×10^{-6} m

Example 10: An optical Fibre has a numerical aperture of 0.20 and a refractive index of cladding is 1.59. Determine the acceptance angle for the Fibre in water which has a refractive index of 1.33.

Solution:

Given

Numerical aperture = 0.20
Refractive index of cladding (n_2) =1.59
Refractive index of water (n_0) = 1.33

Formula: Calculation of refractive index of core

Numerical aperture $= \sqrt{n_1{}^2 - n_2{}^2}$

$$0.20 = \sqrt{n_1{}^2 - (1.59)^2}$$

$$(0.20)^2 = n_1{}^2 - (1.59)^2$$

$$n_1{}^2 = (0.20)^2 + (1.59)^2$$

Refractive index of core (n_1) = 1.60

Calculation of acceptance angle for optical Fibre in water

$$\sin i = \frac{\sqrt{{n_1}^2 - {n_2}^2}}{n_0} = \frac{\sqrt{(1.60)^2 - (1.59)^2}}{1.33}$$

$$\sin i = 0.13$$

$$i = sin^{-1}(0.13)$$

$$\textbf{Acceptance angle } \boldsymbol{i = 7.46^o}$$

Example 11: Relative refractive index of a Fibre is 0.055 and that of core is 1.48. Find numerical aperture, refractive index of cladding, acceptance angle, normalized frequency (V) and number of guided modes when wavelength of light propagated is 1 μm and radius of core is 50μm.

Solution: **Given**

Relative refractive index (Δ) = 0.055

Refractive index of the core (n_1) = 1.48

Wavelength (λ) = 1 μm = 1×10^{-6} m

Radius of the core (a) = 50 μm = 50×10^{-6} m

Formula: Calculation of refractive index of cladding

$$\text{fractinal change inrefractive index } (\Delta) = \frac{n_1 - n_2}{n_1}$$

$$(0.055) = \frac{1.48 - n_2}{1.48}$$

Refractive index of cladding (n_2) = 1.39

Calculation of numerical aperture

$$\text{Numerical aperture} = \sqrt{{n_1}^2 - {n_2}^2}$$

$$= \sqrt{(1.48)^2 - (1.39)^2}$$

Numerical aperture = 0.50

Calculation of acceptance angle

sin (i) = Numerical aperture

sin (i) = 0.50

$$(i) = sin^{-1}(0.50)$$

Acceptance angle (i) = 30°

Calculation of normalized frequency (V)

$$V = \frac{2\pi a}{\lambda}\sqrt{{n_1}^2 - {n_2}^2}$$

$$V = \frac{2\pi \times 50 \times 10^{-6}}{1 \times 10^{-6}} \times \sqrt{(1.48)^2 - (1.39)^2}$$

Normalized frequency (V) = 157

Calculation of number of guided modes

$$\text{Number of guided mode} = \frac{V^2}{2} = \frac{(157)^2}{2}$$

$$= 12324$$

Example 12: A certain optical Fibre has an attenuation of 3.5 dB/km at 850 mm. If 0.5 mW of optical power is initially launched into the Fibre, what is the power level in μW after 4 km?

Solution:

Given

Attenuation $\alpha = 3.5$ dB / km
Initial power level, $P_{in} = 0.5$mW
Length of the cable, L = 4km

Formula: $\alpha = \frac{10}{L}.\log\left(\frac{P_{in}}{P_{out}}\right)$

$$3.5\text{dB/km} = \frac{10}{4km}.\log\left(\frac{0.5mW}{P_{out}}\right)$$

$$3.5\text{dB/km} \times 4\text{km} = 10\ \log\left(\frac{0.5mW}{P_{out}}\right)$$

$$P_{out} = \frac{0.5mW}{25.11} = 19.9\mu\text{W}$$

Example 12: An optical signal has lost 85% its power after traversing 500 m of fibre. What is the loss in dB/km of this fibre?

Solution:

Given

Initial power level, P_{in} = 1mW
Initial power level, P_{out} =85%, , P_{out} =0.85mW
Length of the cable, L = 500m=0.5km

Formula: $\alpha = \frac{10}{L}.\log\left(\frac{P_{in}}{P_{out}}\right)$

$$\alpha = \frac{10}{0.5}\log\left(\frac{1}{0.85}\right)$$

$$\alpha = 1.41 dB/km$$

Short Answer Type Questions

1. Why would you recommend use of optical Fibre in communication system?
2. Explain the principle of total internal reflection.
3. What is the difference between critical angle and angle of acceptance?
4. Describe propagation mechanism of light wave through an optical Fibre.
5. What is meant by numerical aperture and acceptance angle for an optical Fibre?
6. What do mean by step index and graded index Fibre?
7. Why a ray of light takes a zigzag path in a step index Fibre and sinusoidal path in a graded index Fibre?
8. Compare a single mode step index optical Fibre with graded index optical Fibre.
9. What is meant by single mode and multimode optical Fibre?
10. What are advantages of an optical Fibre over conventional communication system?
11. What are different losses in an optical Fibre?
12. What is the unit of measurement for attenuation in OFC?
13. What are the two main types of attenuation in OFC?
14. What is the main cause of intrinsic attenuation in OFC?
15. What is the effect of wavelength on attenuation in Fibre optic cables?

Descriptive Answer Type Questions

1. Derive an expression for numerical aperture of a step index optical Fibre. What is acceptance angle?
2. Derive an expression for numerical aperture in terms of fractional change in refractive index (Δ).
3. Distinguish between mono mode and multimode optical Fibre with labeled diagrams.
4. Differentiate between step-index and graded-index Fibre.
5. Discuss attenuation and dispersion of signal in optical Fibre.
6. Explain with neat sketch optical Fibre communication link.
7. What are the advantages and disadvantages of optical Fibre communication?
8. What are the applications of optical Fibres?
9. Explain the difference between 1D and 2D barcode readers, and provide an example of each.

10. Describe the role of the decoder in a barcode reader, and how it processes barcode data.
11. What are two common applications of barcode readers, and how are they used in each?
12. Explain the causes and effects of attenuation in Optical Fibre Communication (OFC).
13. Describe the difference between intrinsic and extrinsic attenuation in OFC. Provide examples of each.
14. Discuss the methods to minimize attenuation in OFC systems.

CHAPTER **3.**

Interference in Thin Film

Interference by division of amplitude, Interference in thin film of constant thickness due to reflected and transmitted light; origin of colors in thin film; Wedge shaped film; Newton's rings. Applications of interference - Determination of thickness of very thin wire or foil; determination of refractive index of liquid; wavelength of incident light; radius of curvature of lens; testing of surface flatness; Anti-reflecting films and Highly reflecting film.

3.1. INTRODUCTION

The properties of light and its behavior on interacting with matter are being used in various scientific and engineering applications. It is imperative to understand some of the features of light that form the basis for such applications. The branch of physics devoted to the study of light is named as Optics. To elucidate different aspects the study is classified into geometric optics, physical optics and modern optics.

(a) Geometric optics or ray optics: Geometric optics studies the position of the image formed with a lens or mirror and the calculation of magnification of image. This part of optics explains the lens formula, mirror formula and the phenomena like reflection, refraction as well as dispersion of light assuming the rectilinear propagation of light.

(b) Physical optics: More precise study shows that the results obtained on the assumption that light travels in a straight line are only approximate. The physical optics, also known as wave optics, gives emphasis on the wave nature of light to explain the phenomena like interference, diffraction and polarization of light.

(c) Modern optics: It covers the study of transmission of light through optical fiber (Fiber Optics), and light amplification by stimulated emission of radiation (LASER). In this chapter we will discuss the phenomenon of interference of light in thin films and some of its applications.

3.2 INTERFERENCE

The phenomenon of interference of light waves is the direct consequence of Huygen's principle of superposition of waves applicable to water waves, mechanical waves, sound waves and electromagnetic waves.

The **principle of superposition of waves** states that when two or more waves move simultaneously through a region of space, each wave advances independently as if other waves were not present. The resulting wave displacement at any point and time is the vector sum of the displacements of the individual waves.

The **interference of light waves** can be defined as: *"when two light waves cross each other there is addition of amplitudes if the crest of one meets the crest of another wave and if the crest of one wave meets the trough of another the resultant is subtraction of amplitudes".*

The colour effects of thin films are due to interference of light waves (the beats produced by two notes of similar frequency are the result of the interference of sound waves).

If the resultant amplitude is sum of the amplitudes due to two interfering waves the interference is known as constructive interference and if the resultant amplitude is the difference of two amplitudes it is called destructive interference as shown in fig. 3.1 (a) and 3.1 (b).

The phenomenon of interference in case of light waves was first observed by a British physicist Thomas Young in 1801.

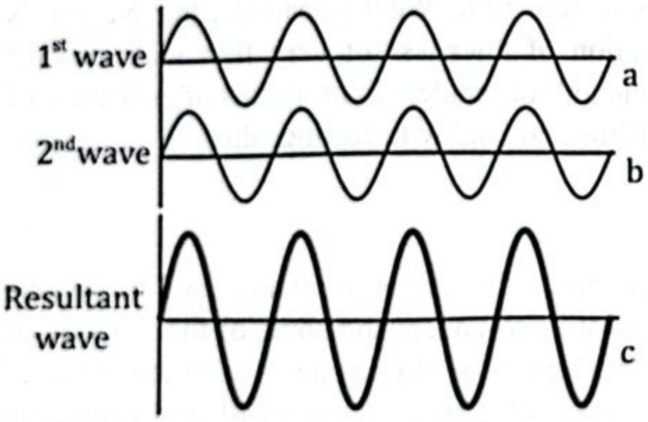


Fig: 3.1 (a)

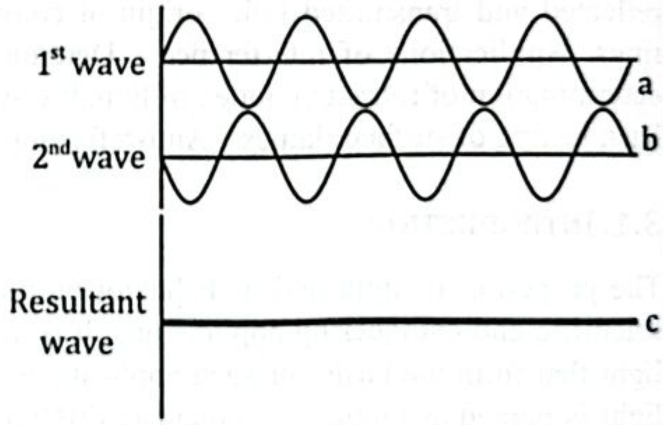


Fig: 3.1 (b)

3.3 Conditions to Obtain Sustainable Interference

(a) The light waves from the sources must have same frequency and be coherent.

(b) The coherent sources must be close to each other.

(c) The amplitudes of the light waves must be same for good contrast between bright and dark fringes.

(d) The path difference between the light waves must be less than the coherence length of the waves.

(e) For plane polarized waves, their plane of polarization must be same.

3.4 Methods of Producing Interference

An interference pattern can be obtained either by (a) division of wavefront or (b) division of amplitude.

(a) **Interference by division of wavefront:** In this method the beam of light waves from a single source is physically divided into two parts using a pair of slits or a mirror or a biprism.

(b) **Interference by division of amplitude:** In this method the beam of light waves from a source is optically divided by partial reflection and refraction from two surfaces.

This way the amplitude of the beam incident on a thin transparent film is divided into two parts.

3.5 Thin Film

A film is said to be a thin film if the thickness of the film is of the order of wavelength of visible light which is taken to be 5500 Å. If the thickness of the film is about 10- 50 μm it is said to be a thick film. A soap bubble, an oil slick on a puddle of water, a thin glass or mica sheet, a film deposited on a glass plate, or a thin air or liquid film enclosed between two transparent glasses sheets are the examples of thin films. When light falls on thin film, part of it undergoes reflection and part of it undergoes refraction.

3.6 Change of Phase on Reflection

The points in the path of a wave motion are said to be points of equal phase if the displacements at those points at any moment are exactly same (i.e. of the same magnitude and varying in the same manner).

The light wave when gets reflected back into the optically rarer medium from a boundary between an optically rarer and denser medium it suffers a phase change of 180° or π radians (equal to path difference of $\lambda/2$). However there is no such phase reversal when it is reflected back into the optically denser medium from a boundary between optically denser and rarer medium. Also no phase change occurs in case of transmission from rarer to denser or denser to rarer medium as shown in Fig. 3.2(a).

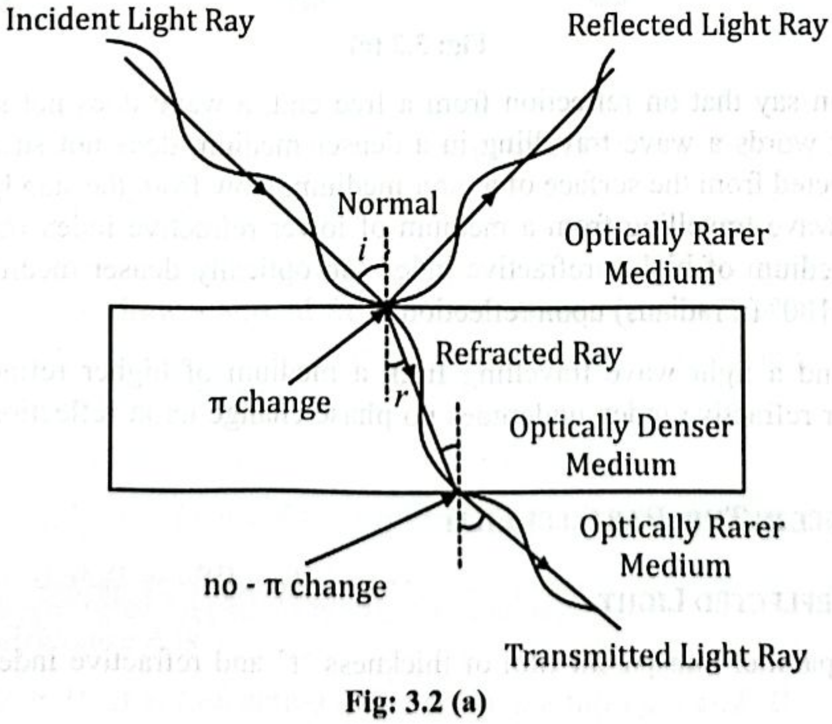


Fig: 3.2 (a)

To understand the change of phase of light waves when they are reflected from a boundary let us recall behavior of wave on a string. Suppose one end of the stretched string is tied to a rigid support and a pulse created travels along the length of the string (rarer medium) towards the end fixed on the rigid support (denser medium). As the pulse arrives at that end, it exerts an upward force on the support. However, the support is rigid and does not move.

According to Newton's third law of action and reaction, the support exerts an equal but oppositely directed force on the string. This force produces a pulse which is reflected with its transverse displacement reversed Fig. 3.2 (b).

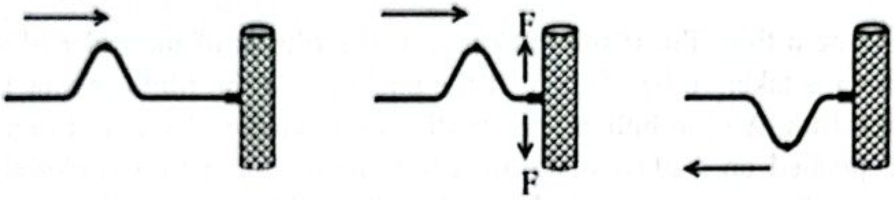


Fig: 3.2 (b)

From this we can say that on reflection from a fixed end, a wave suffers an abrupt phase change of 180° or π radians. In other words a wave travelling in a rarer medium suffers a reversal of phase when reflected from the surface of a denser medium.

Next, consider the string (denser medium) attached to a very light ring (rarer medium) which is free to slide along a rod without friction. A pulse created, when arrives at the end attached to the ring, the ring starts sliding down and generates a pulse which reflects with its transverse displacement in the same direction as that of the incident pulse Fig. 3.2 (c).

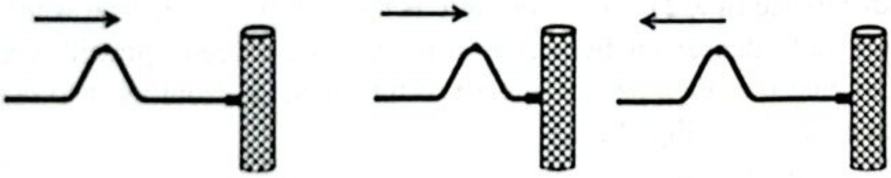


Fig: 3.2 (c)

From this we can say that on reflection from a free end, a wave does not suffer any phase change. In other words a wave travelling in a denser medium does not suffer a reversal of phase when reflected from the surface of a rarer medium. Now from the standpoint of physical optics the light wave travelling from a medium of lower refractive index (an optically rarer medium) to a medium of higher refractive index (an optically denser medium) undergoes a phase change of 180° (π radians) upon reflection.

On the other hand a light wave travelling from a medium of higher refractive index to a medium of lower refractive index undergoes no phase change upon reflection. This is shown in Fig. 3.2 (a).

3.7 Interference in Thin Parallel Film

3.7.1 Due to Reflected Light

Consider a thin parallel transparent film of thickness 't' and refractive index μ as shown in Fig. 3.3.

A ray AB of monochromatic light of wavelength λ incident upon the upper surface of this film at an angle of incident 'i' will be partially reflected along BC and partially refracted along BD. The refracted ray BD is partially reflected along DF in the film and partially transmitted along DE. Similarly the ray DF undergoes multiple reflection and refraction along FH and FG respectively.

Since the rays on each side have been derived from the same incident ray AB they are coherent. However, they travel different distances and therefore have path differences among themselves. Due to this path difference these rays produce interference patterns.

Let us consider the reflected rays BC and FG and determine the path difference between them.

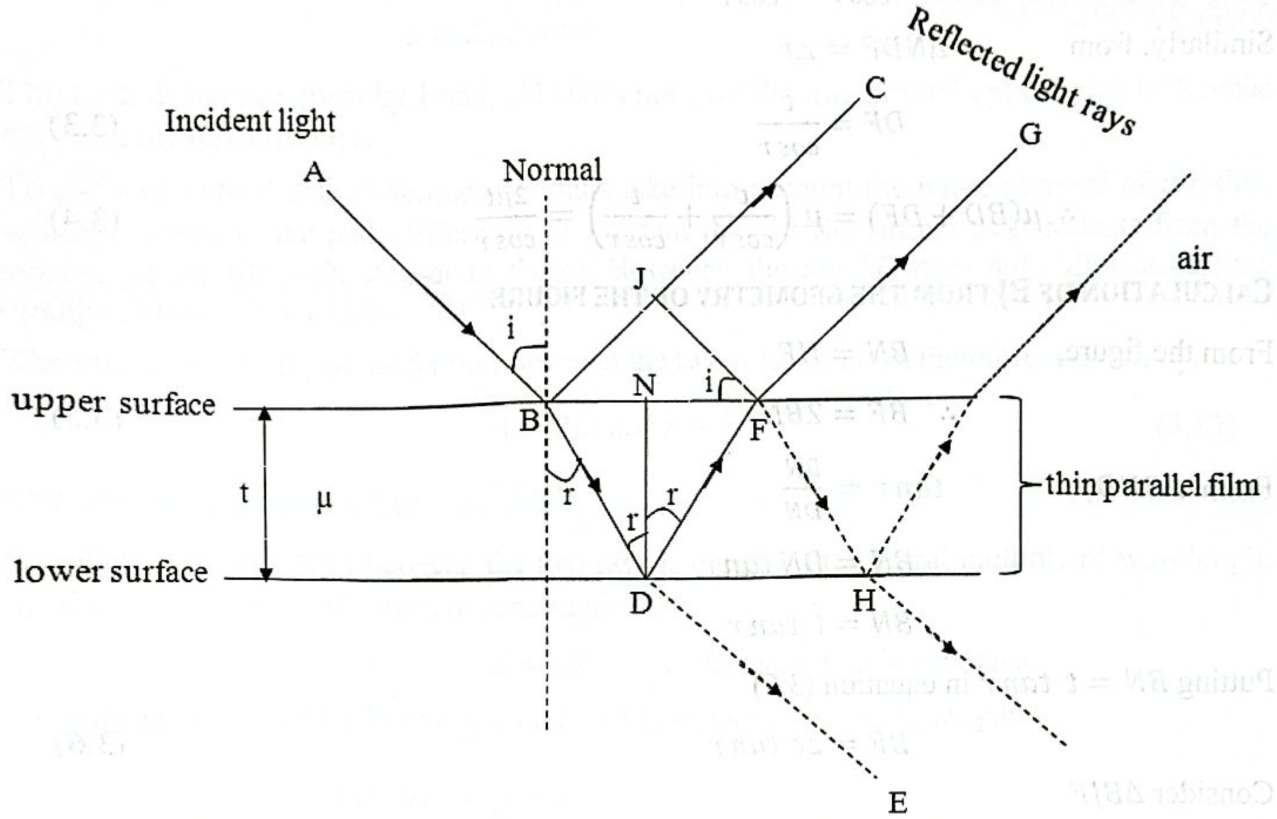


Fig: 3.3 Interference in thin transparent parallel film by reflected system

GEOMETRIC PATH DIFFERENCE

Consider the ray ABC and ray ABDFG. In order to determine the geometric path difference between these two rays, draw a normal from point F to the ray BC. These two reflected rays have been derived from ray AB therefore upto point B they travel the same distance and beyond JF they travel equal distance.

Hence the geometric path difference between them is given by.

$$G.P.D. = (\text{Path of the rays in thin film}) - \text{Path of the rays in air}$$

$$\therefore\ G.P.D. = (BD + DF) - BJ$$

The optical path difference Δ is

$$O.P.D.\ \Delta = \text{Refractive index of the medium } \mu \times G.P.D.$$

$$\Delta = \mu(BD + DF) - 1 \times BJ \quad [\because \mu_{air} = 1] \tag{3.1}$$

CALCULATION OF BD AND DF FROM THE GEOMETRY OF THE FIGURE

In ΔBDF, $\quad \angle BDN = \angle NDF = \angle r$

$$BD = DF$$

From, $\angle BDN = \angle r$

$$\cos r = \frac{DN}{BD}$$

$$BD = \frac{DN}{\cos r} = \frac{t}{\cos r} \tag{3.2}$$

Similarly, from $\quad \Delta NDF = \angle r$

$$DF = \frac{t}{\cos r} \tag{3.3}$$

$$\therefore \mu(BD + DF) = \mu\left(\frac{t}{\cos r} + \frac{t}{\cos r}\right) = \frac{2\mu t}{\cos r} \tag{3.4}$$

CALCULATION OF BJ FROM THE GEOMETRY OF THE FIGURE

From the figure, $\quad BN = NF$

$$\therefore \quad BF = 2BN \tag{3.5}$$

From ΔBND, $\quad \tan r = \frac{BN}{DN}$

$$BN = DN \tan r$$

$$BN = t \tan r$$

Putting $BN = t \tan r$ in equation (3.5)

$$BF = 2t \tan r \tag{3.6}$$

Consider ΔBJF

$$\sin i = \frac{BJ}{BF}$$

$$BJ = BF \sin i \tag{3.7}$$

Put $BF = 2t \tan r$ from equation (3.6) in equation (3.7)

$$BJ = 2t \tan r \sin i \tag{3.8}$$

By Snell's law, $\quad \mu = \frac{\sin i}{\sin r}$

$$\therefore \quad \sin i = \mu \sin r \tag{3.9}$$

Put $\sin i = \mu \sin r$ in equation (3.8)

$$BJ = 2t \tan r \, \mu \sin r$$

$$BJ = 2\mu t \frac{\sin r}{\cos r} \sin r$$

$$BJ = 2\mu t \frac{\sin^2 r}{\cos r} \tag{3.10}$$

Put equation (3.4) and (3.10) in equation (3.1),

$$\text{Optical path difference } (\Delta) = \frac{2\mu t}{\cos r} - \frac{2\mu t \sin^2 r}{\cos r}$$

$$= \frac{2\mu t}{\cos r}(1 - \sin^2 r)$$

$$\Delta = 2\,\mu t \cos r \qquad (3.11)$$

The path difference given by Eqn.(3.11) does not give the true or total optical path difference between the reflected rays.

To get true optical path difference we must take into account the phase reversal of π radian which is equal to the path difference of λ/2 that the ray BC suffers as it reflects from the surface of the film, (the denser medium). However, the ray FG does not suffer any phase change at film- air boundary.

The effective optical path difference between the two reflected rays therefore is given by,

$$\Delta = 2\mu t \cos r - \frac{\lambda}{2} \qquad (3.12)$$

CONDITION FOR MAXIMA (BRIGHTNESS)

If optical path difference between the two rays is equal to an integral multiple of wavelength λ, then the two rays will interfere constructively,

$$\Delta = n\lambda \text{ .... Constructive interference}$$

By putting the value of Δ from equation (1.1.12) in above condition, we get,

$$2\mu t \cos r - \frac{\lambda}{2} = n\lambda$$

$$\therefore \quad 2\mu t \cos r = (2n+1)\frac{\lambda}{2} \qquad n = 0, 1, 2, 3, \ldots.. \qquad (3.13)$$

If Eqn. (3.13) is satisfied then the film will appear ***bright*** in the reflected light.

CONDITION FOR MINIMA (DARKNESS)

If optical path difference between the two rays is odd integral number of half wavelength (λ/2) then the two rays will interfere destructively.

$$\Delta = (2n+1)\frac{\lambda}{2} \quad \text{...... Destructive interference}$$

By putting the value of Δ from Eqn. (3.12) in above condition, we get,

$$2\mu t \cos r - \frac{\lambda}{2} = (2n+1)\frac{\lambda}{2}$$

$$2\mu t \cos r = (n+1)\lambda$$

If one full wave is added to or subtracted from any of interfering waves it does not affect on optical path difference between the rays. Therefore $(n+1)\lambda$ is replaced by $n\lambda$, hence the above condition can be written as,

$$\therefore \quad 2\mu t \cos r = n\,\lambda \qquad n = 1, 2, 3, \ldots.. \qquad (3.14)$$

If Eqn. (3.14) is satisfied then the film will appear ***dark*** in the reflected light.

3.7.2 DUE TO TRANSMITTED LIGHT

Consider a thin transparent parallel film of thickness t and refractive index μ as shown in (Fig 3.4) When a ray AB of monochromatic light of wavelength λ is incident on upper surface of this thin parallel film at an angle of incidence i, it will be partially refracted along BC. The ray BC will then be partially reflected along CE and partially transmitted along CD. The ray CE will again be transmitted along FG.

In order to calculate the optical path difference between transmitted rays, draw EM and FN normal on CF and CD, respectively.

The optical path difference between transmitted rays CD and FG is given by,

$$O.P.D.\ \Delta = \text{Path of the rays in thin film} - \text{Path of the rays in air}$$

$$\Delta = \mu(CE + EF) - \mu_{air} \times (CN) \qquad (3.15)$$

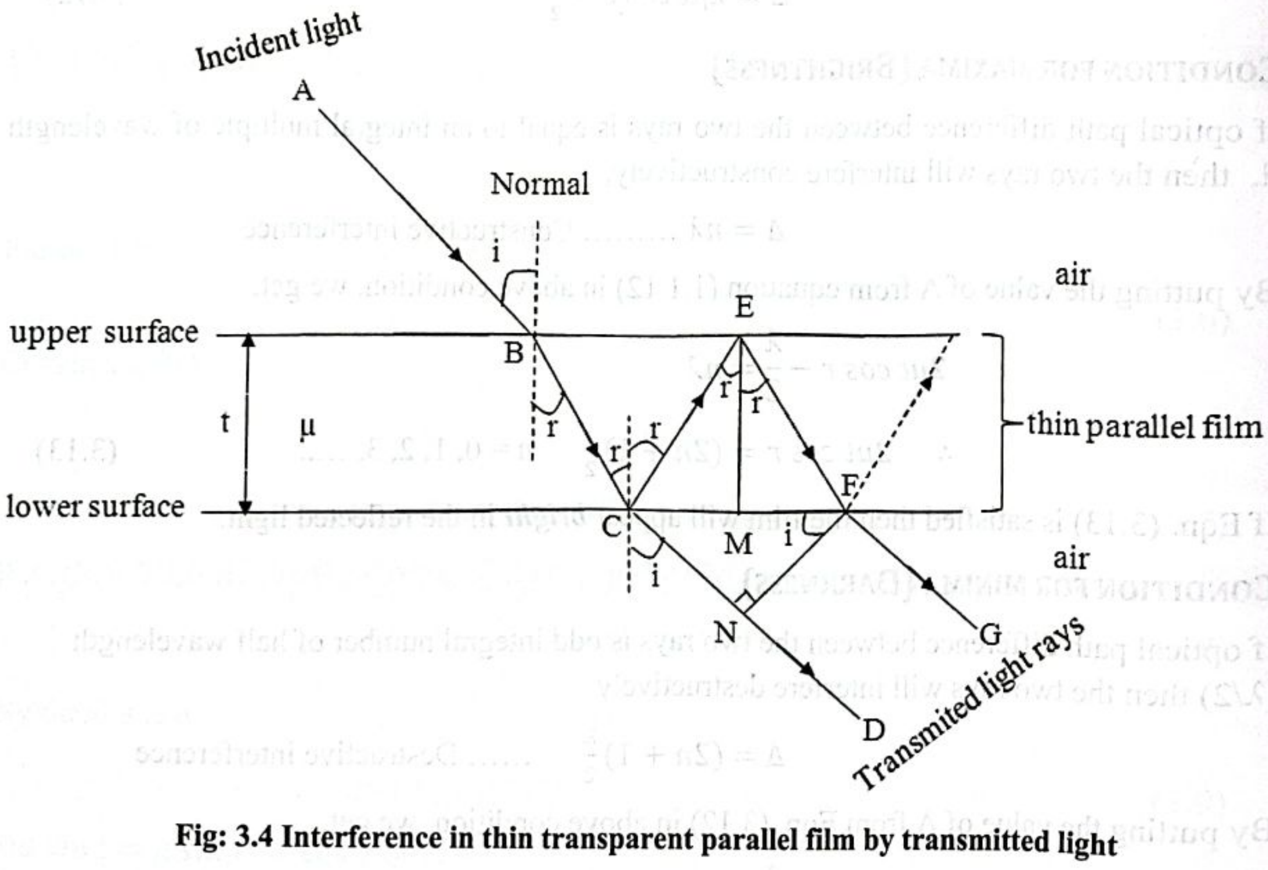


Fig: 3.4 Interference in thin transparent parallel film by transmitted light

From ΔCME,

$$\cos r = \frac{EM}{CE} = \frac{t}{CE}$$

$$\therefore CE = \frac{t}{\cos r}$$

Similarly from ΔFME,

$$EF = \frac{t}{\cos r}$$

Thus, $$\mu(CE + EF) = \frac{2\mu t}{\cos r} \quad (3.16)$$

From geometry of the figure,

$$CM = MF$$

$$\therefore CF = 2CM \quad (3.17)$$

From ΔCME, $$\tan r = \frac{CM}{ME} = \frac{CM}{t}$$

$$\therefore CM = t \tan r$$

Using CM in Eqn. (3.17),

$$CF = 2t \tan r \quad (3.18)$$

Considering ΔCNF, $$\sin i = \frac{CN}{CF}$$

$$CN = CF \sin i$$

Using Eqn. (3.18) we get,

$$CN = 2t \tan r \sin i$$

$$CN = 2t\,\frac{\sin r}{\cos r}\,\mu \sin r \qquad \left(\because \text{By Snell's law, } \mu = \frac{\sin i}{\sin r}\right)$$

$$CN = 2\mu t\,\frac{\sin^2 r}{\cos r} \quad (3.19)$$

Putting equation (3.16) and (3.19) in equation (3.15), we get

$$\text{Optical Path Difference } (\Delta) = \frac{2\mu t}{\cos r} - \frac{2\mu t \sin^2 r}{\cos r}$$

$$= \frac{2\mu t}{\cos r}\,(1 - \sin^2 r)$$

$$\Delta = 2\,\mu t \cos r \quad (3.20)$$

The light rays CD and FG are transmitted from thin film (optically denser medium) to air (rarer medium), hence these rays do not suffer any phase change. Therefore equation (3.20) gives true optical path difference for transmitted rays.

CONDITION FOR MAXIMA (BRIGHTNESS)

If optical path difference between the two rays is equal to an integral multiple of wavelength λ then the two rays will interfere constructively,

$$\Delta = n\lambda$$

By putting the value of Δ from equation (3.20) in above condition, we get,

$$2\,\mu t \cos r = n\lambda \qquad n = 1,2,3,... \tag{3.21}$$

If Eqn. (3.21) is satisfied, the film will appear ***bright*** in transmitted light.

CONDITION FOR MINIMA (DARKNESS)

If optical path difference between the two rays is odd integral number of half wavelength $\lambda/2$ then the two rays will interfere destructively,

$$\Delta = (2n+1)\frac{\lambda}{2}$$

By putting the value of Δ from equation (3.20) in above condition, we get,

$$2\,\mu\, t\, \cos r = (2\,n+1)\frac{\lambda}{2} \qquad n = 0, 1, 2,... \tag{3.22}$$

If Eqn. (3.22) is satisfied, the film will appear ***dark*** in the transmitted light. In the case of transmitted light the interference fringes obtained are less distinct because the difference in amplitude between CD and FG is very large.

From equations (3.13), (3.14), (3.21) and (3.22) it is observed that the conditions of minima and maxima are just reverse of the conditions for reflected light. Therefore the thin film which will appear **bright in reflected light** appears **dark in transmitted light** and vice versa.

3.8 COLOURS IN THIN FILMS

When white light is incident on a thin film, the reflected light will not include the colour whose wavelength satisfies the condition $2\mu t \cos r = n\lambda$. The film will appear coloured if wavelength satisfies the condition $2\mu t \cos r = (2n+1)\frac{\lambda}{2}$ and it depends on the thickness of the film, the refractive index of the film, and the angle of incidence of light rays.

(a) Suppose incident light is white and thin film is parallel so that thickness of the film t and angle of refraction r are constant. The path difference is different for each colour because refractive index μ is different for different colours. Hence we will see different colours with increasing wavelengths.

(b) If thin film is parallel but incident white light is not parallel, then the path difference will vary due to change in angle of refraction r, hence thin film shows different colours when observed from different directions.

(c) If the incident light is white and parallel but the thin film is of varying thickness then also different colours are seen.

3.9 WEDGE SHAPED AIR FILM (NON-UNIFORM THICKNESS FILM)

A thin wedge of air film can be obtained by placing two plane glass plates in contact at one edge and separated by a thin sheet of paper at the opposite end. Let us consider two plane glass plates A and B. They are kept in contact at the point O and are separated at other edge by introducing thin sheet of paper or thin wire as shown in Fig. 3.5.

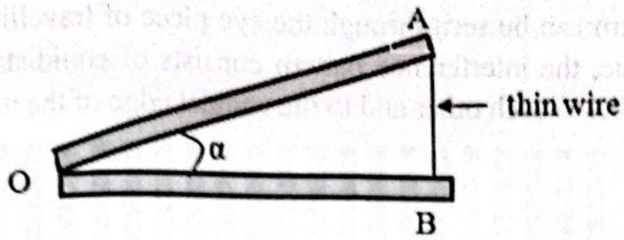


Fig: 3.5 Wedge shaped air film

The air film of wedge shape is enclosed between two plane glass plates. The thickness of the air film is zero at the point of contact of two glass plates and gradually increases from point of contact to the other end i.e. maximum at other end. The wedge shaped air film is used in an experiment to determine the thickness of a thin sheet of paper or thin wire and testing of optical flatness of plane glass plate.

EXPERIMENTAL ARRANGEMENT OF WEDGE SHAPED AIR FILM

In the experimental setup of wedge shaped air film a parallel beam of monochromatic light from source 'S' is made to incident on the glass plate G kept inclined at an angle of 45° to the direction of light as shown in Fig. 3.6.

The rays reflected from the glass plate 'G' are incident normally on the wedge shaped air film enclosed between two plane glass plates. A part of the incident light is partially reflected from the lower surface of upper glass plate and rest of the light is transmitted through it. The part of the transmitted light is partially reflected from the upper layer of lower glass plate. These two reflected rays are obtained by the division of amplitude from an incident ray of light. The two reflected rays interfere constructively or destructively depending upon the path difference between them.

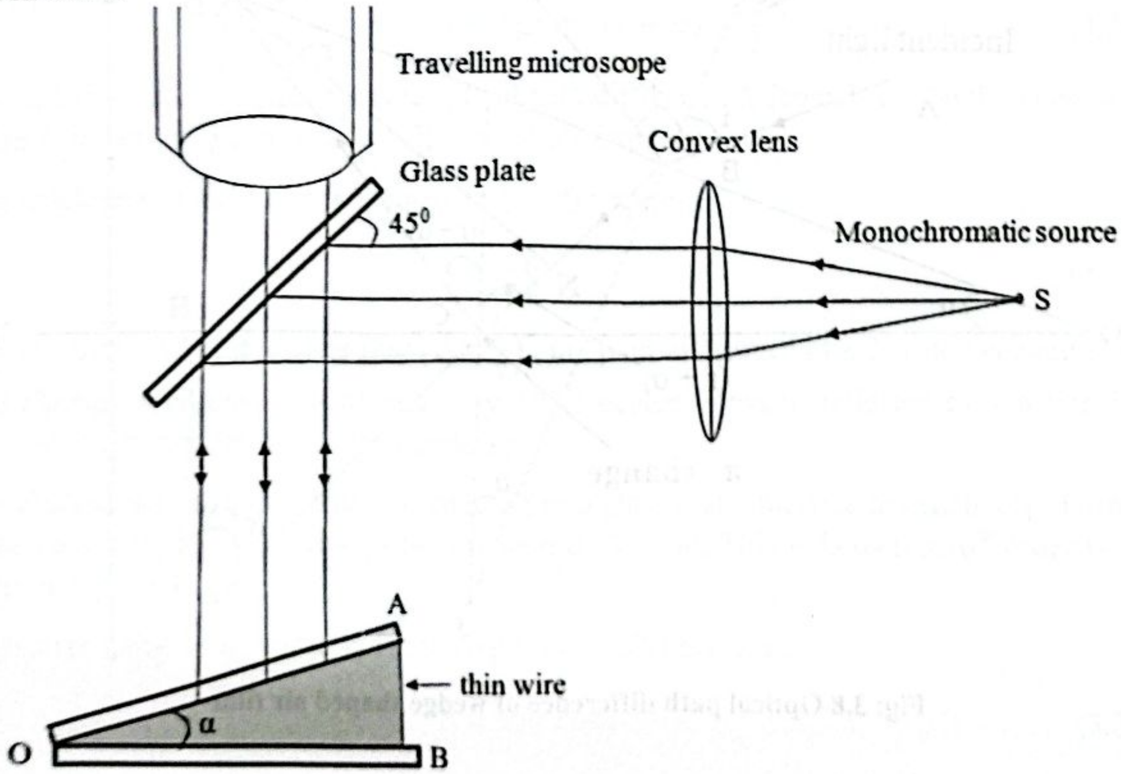


Fig: 3.6 Experimental set-up of wedge shaped air film

The interference pattern can be seen through the eye piece of travelling microscope. If incident light is monochromatic, the interference pattern consists of equidistant, straight alternate dark and bright bands parallel to each other and to the contact edge of the wedge as shown in Fig. 3.7.

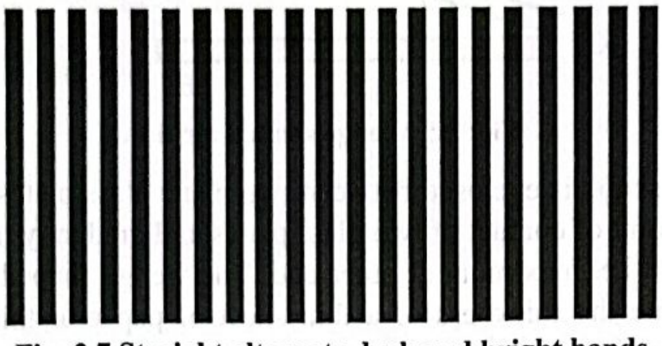


Fig: 3.7 Straight alternate dark and bright bands

In case of white light, colored fringes are seen.

The interference fringe pattern always begins with dark fringe because the thickness of air film is zero at the point of contact. This is known as **zero**th **order** band.

CALCULATION OF OPTICAL PATH DIFFERENCE

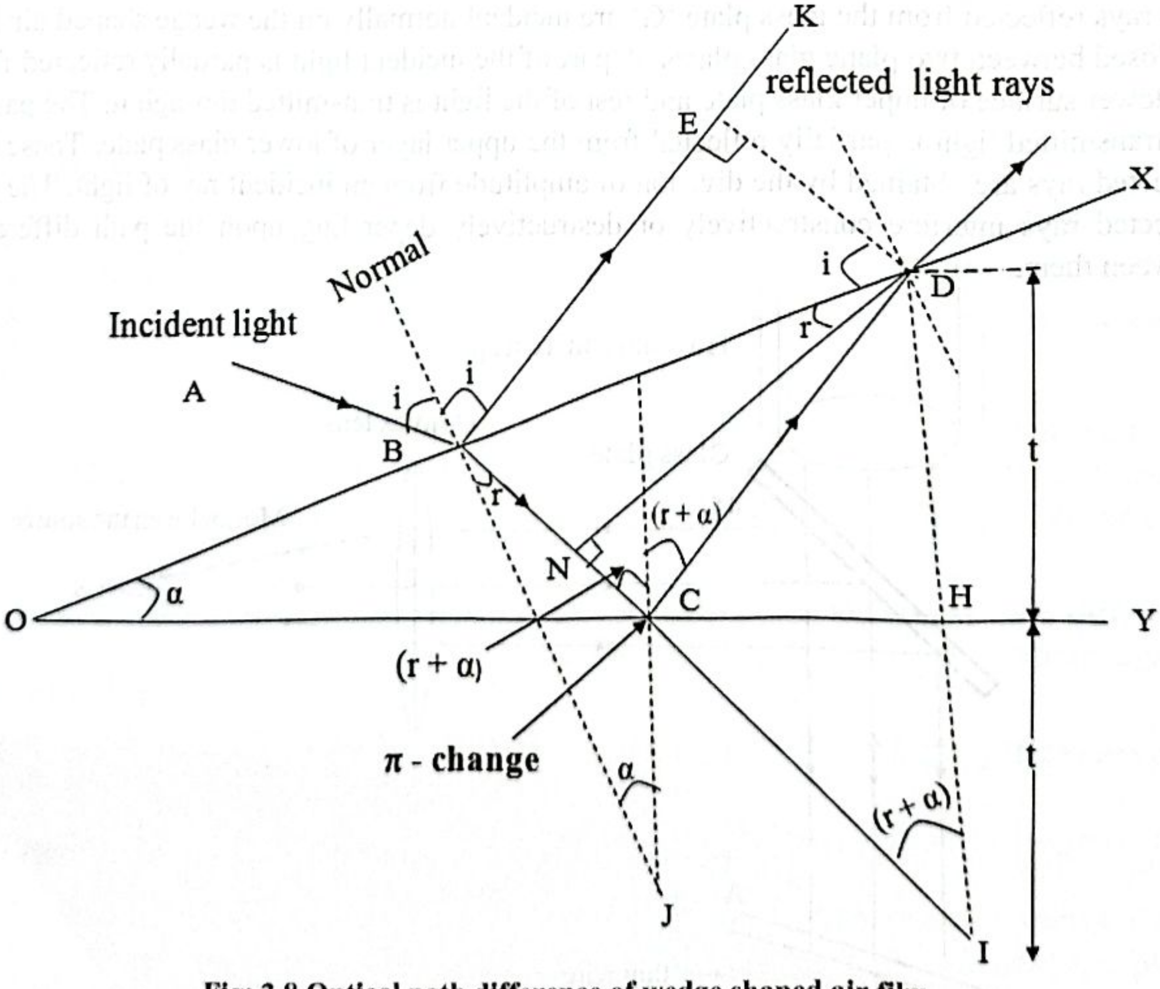


Fig: 3.8 Optical path difference of wedge shaped air film

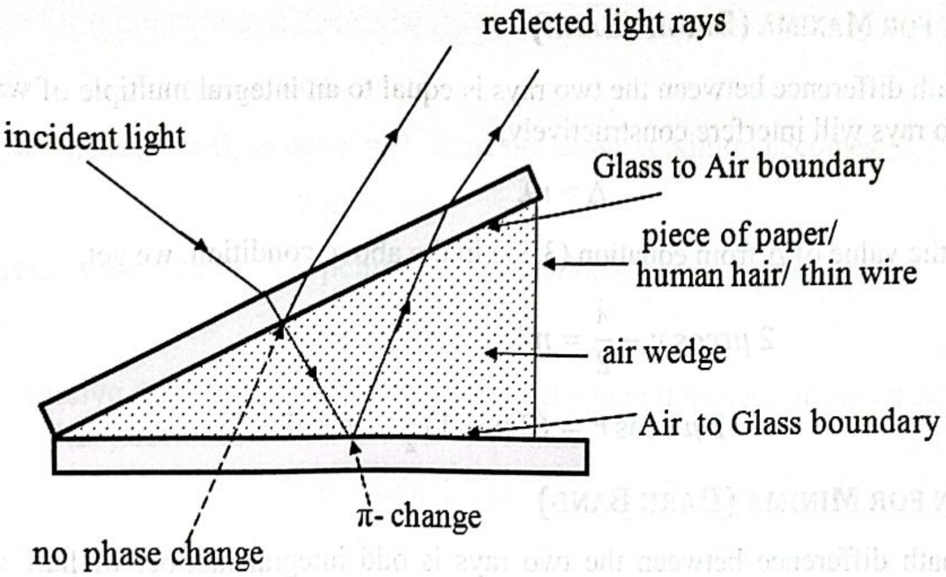


Fig: 3.9 Interference in wedge shaped air film.

Consider two glass plates OX and OY inclined at an angle of wedge α as shown in Fig. (3.8). A beam of monochromatic light AB is incident obliquely on the upper surface of the film. The ray AB is partially reflected along BK and rest of it refracted along BC, the refracted ray is partially reflected along CD. Similarly ray CD undergoes multiple reflections and is also refracted along DL.

The interference takes place between the reflected rays BK and DL because they are obtained by the division of amplitude from the incident ray AB. The path difference between the two reflected rays BK and DL is given by,

$$\Delta = 2\mu t\, cos(r + \alpha) - \frac{\lambda}{2} \qquad (3.29)$$

From equation (3.29) it clear that the optical path difference Δ depends on the thickness of the air film t, however thickness of air film is not uniform.

At the thickness of air film $t = 0$, equation (3.29) becomes

$$\Delta = -\frac{\lambda}{2} \qquad (3.30)$$

From equation (3.30) it is clear that $\left(\Delta = \frac{\lambda}{2}\right)$, the path difference of λ/2 which is equivalent to phase change π radians (out of phase by 180°) occurs between reflected rays which is the condition for minimum intensity (darkness).

The reflected waves from point of contact of two glass plate interfere destructively. Therefore interference fringe pattern always begins with dark band. This is known zeroth order band as shown in Fig. (3.7).

For smaller value of α, $\cos\alpha = 1$, then equation (3.29) becomes

$$\Delta = 2\mu t\, cos\, r - \frac{\lambda}{2} \qquad (3.31)$$

The equation (3.31) gives the optical path difference between the reflected rays.

Condition for Maxima (Bright Band)

If optical path difference between the two rays is equal to an integral multiple of wavelength λ then the two rays will interfere constructively,

$$\Delta = n\lambda$$

By putting the value of Δ from equation (3.31) in the above condition, we get,

$$2\,\mu t \cos r - \frac{\lambda}{2} = n\lambda$$

$$2\,\mu\,t\cos r = (2\,n+1)\frac{\lambda}{2} \qquad n = 0, 1, 2,\ldots \qquad (3.32)$$

Condition for Minima (Dark Band)

If optical path difference between the two rays is odd integral number of half wavelength (λ/2) then the two rays will interfere destructively,

$$\Delta = (2n+1)\frac{\lambda}{2}$$

By putting the value of Δ from equation (3.31) in the above condition, we get,

$$2\mu t\, cos\, r - \frac{\lambda}{2} = (2n+1)\frac{\lambda}{2}$$

$$2\mu t\, cos\, r = (n+1)\lambda$$

If one full wave is added or subtracted from any interfering waves then it does not affect the optical path difference between the rays. Therefore $(n+1)\lambda$ is replaced by $n\lambda$, hence above condition can be written as,

$$2\,\mu t\cos r = n\lambda \qquad n = 1,2,3,\ldots \qquad (3.33)$$

Calculation of Fringe Width (β)

The distance between successive dark fringes or bright fringes is known as fringe width (β).

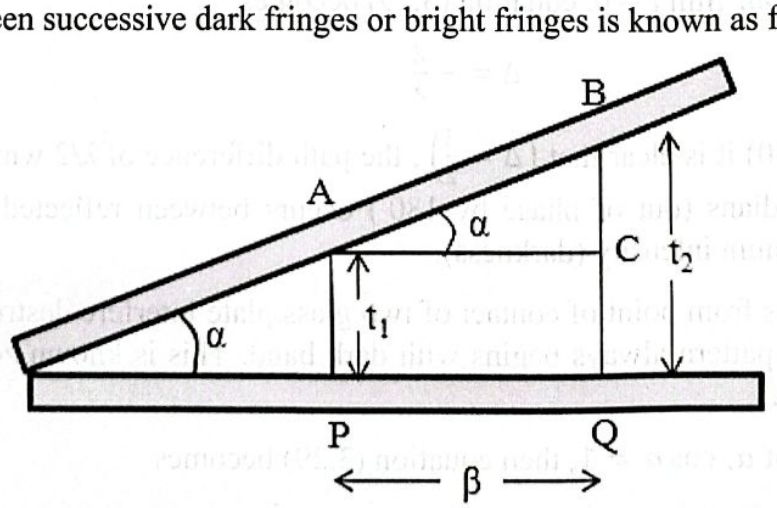


Fig: 3.10 Calculation of fringe width of wedge shaped air film

Points A and B are the positions of two successive dark fringes. The corresponding film thickness are AP = t_1 and BQ = t_2, respectively as shown in Fig. (3.10).

The condition for minima or dark fringe is,

$$2\,\mu\, t \cos r = n\lambda$$

For normal incidence, $r = 0$, so $\cos r = 1$, thus, the above condition becomes,

$$2\,\mu t = n\lambda \tag{3.34}$$

If the thickness of the film $t = t_1$ at point A, then equation (3.34) becomes

$$2\mu t_1 = n\lambda \tag{3.35}$$

The next successive dark fringe will occur at point B where thickness of the air film is $t = t_2$, then equation (3.34) becomes

$$2\,\mu t_2 = (n+1)\lambda \tag{3.36}$$

Subtracting Eqn. (3.35) from Eqn. (3.36)

$$2\,\mu(t_2 - t_1) = \lambda$$

$$2\,\mu(BC) = \lambda \qquad (\because\ t_2 - t_1 = BC\,)$$

$$\therefore\ BC = \frac{\lambda}{2\mu} \tag{3.37}$$

From $\Delta(ACB)$, $\angle BAC = \alpha$

$$tan\,\alpha = \frac{BC}{AC}$$

$$BC = AC \tan\alpha$$

Put $BC = AC\ tan\ \alpha$ in equation (3.37)

$$AC\ tan\alpha = \frac{\lambda}{2\mu}$$

AC is the distance between two successive dark fringes and is known as fringe width (β)

$$\beta = \frac{\lambda}{2\mu\ tan\ \alpha}$$

As, α is very small, $\tan\alpha \approx \alpha$

So, Fringe width $$\beta = \frac{\lambda}{2\mu\,\alpha} \tag{3.38}$$

For air film ($\mu_{air} = 1$), therefore Eqn. (3.38) becomes,

$$\beta = \frac{\lambda}{2\,\alpha} \tag{3.39}$$

DETERMINATION OF THICKNESS OF THIN SHEET OF PAPER

A thin sheet of paper is used to obtain wedge shaped air film between the two glass plates as shown in Fig. (3.11).

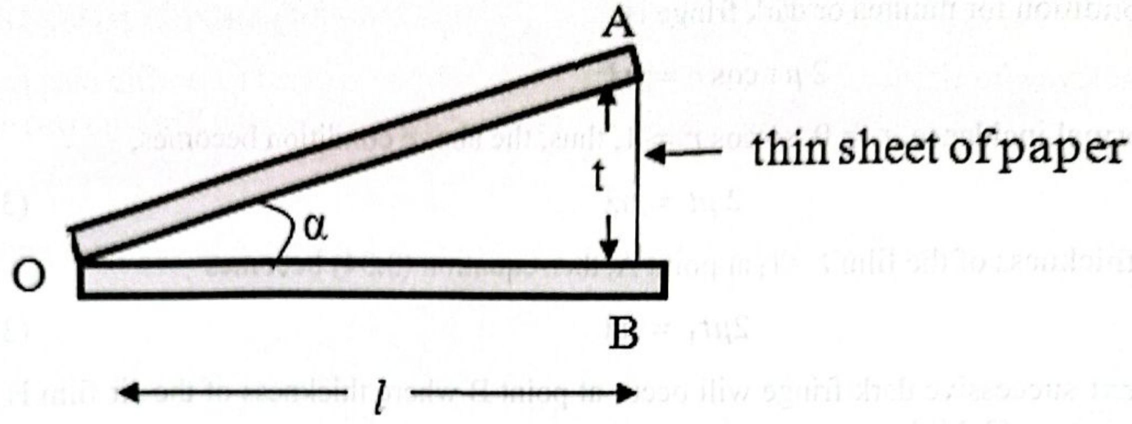


Fig: 3.11 Calculation of thickness of wedge shaped air film

Let α be the wedge angle, t the thickness of a thin sheet of paper, and l the length of the air wedge. From the geometry of Fig. (3.11)

$$tan\ \alpha = \frac{t}{l}$$

If α is very small then $tan\ \alpha \approx \alpha$

$$\therefore \alpha = \frac{t}{l} \tag{3.40}$$

Equation (3.39) can be rearranged as

$$\alpha = \frac{\lambda}{2\mu\beta} \tag{3.41}$$

From equation (3.40) and (3.41)

$$\frac{t}{l} = \frac{\lambda}{2\mu\beta}$$

$$t = \frac{\lambda l}{2\mu\beta} \tag{3.42}$$

Equation (3.42) gives the thickness of a thin sheet of the paper.

3.10 Newton's Rings in Reflected Light

When a plano-convex lens of large radius of curvature is placed on a plane glass plate such that its curved surface lies on a plane glass plate then an air film of gradually increasing thickness is formed between the plano-convex lens and the plane glass plate. If monochromatic light is allowed to fall normally on the thin air film and the film is viewed in reflected light, alternate bright and dark concentric circular rings with increasing radii are observed around the point of contact (Fig. 3.13).

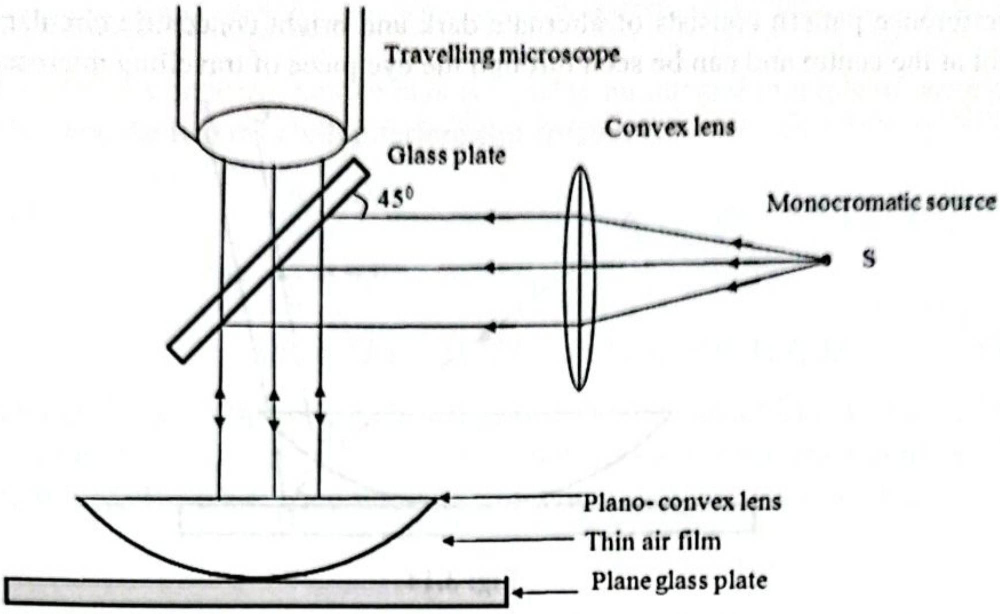


Fig: 3.12 Experimental set-up of Newton's rings

The interference fringes so formed were first observed by Hooke but described by Newton and hence are known as Newton's rings. Newton's rings are used in laboratory for the determination of wavelength of monochromatic light and refractive index of a liquid.

As shown in Fig. (3.12) a plano-convex lens of large radius of curvature R is placed on a plane glass plate with its convex surface resting on the plate. A thin air film is enclosed between the plano-convex lens and plane glass plate having thickness t zero at the point of contact and gradually increasing outward. The monochromatic light from an extended source such as sodium lamp falls on a glass plate G held at an angle 45° with the direction of the rays.

The rays reflected from the glass plate G are incident normally on the air film enclosed between the plano-convex lens and plane glass plate. The incident light is partially reflected from the upper and lower surfaces of the air film and rest is transmitted through it.

Fig: 3.13 Newton's rings by reflected light (centre of the system is dark)

The reflected rays from the top and bottom surfaces of the air film superimpose upon each other with the path difference depending on the thickness of the air film at the point of incidence and gives rise to an interference pattern.

The interference pattern consists of alternate dark and bright concentric circular rings with a dark spot at the centre and can be seen through the eye piece of travelling microscope.

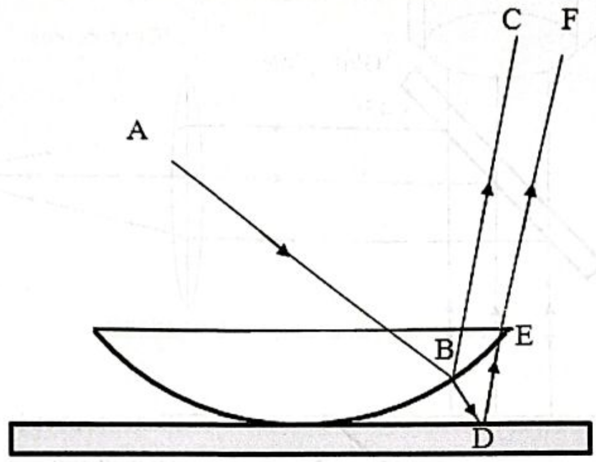


Fig: 3.14

As shown in Fig. (3.14) the incident ray AB is partially reflected at point B and goes along BC while rest part of it is transmitted through the air film. The transmitted ray BD is partially reflected at point D and goes along DEF. The ray BC is reflected from the bottom of the plano-convex lens and so there is no phase reversal.

However the transmitted ray BD undergoes a phase change of π radians on reflection at point D because it reflects from upper surface of a plane glass plate which is an optically denser medium compared to an air film (or liquid film if air is replaced by liquid).

Let us find the path difference between two rays BC and BDEF. If points B and E are assumed to be very close to each other, then from these points onwards, the two rays travel equal paths. Thus the extra path travelled by ray BDEF is (BD +DE). Further, let us make two assumptions

(a) BD = DE, and

(b) BD = t i.e. the thickness of the air film at point B

Also, although in the Fig. (3.14) oblique incidence is shown for sake of explanation, for normal incidence angle of refraction

$$r = 0 \text{ or } \cos r = 1.$$

The optical path difference between the two interfering rays is therefore,

$$O.P.D.\ \Delta = 2BD = 2t \text{ .... in air film}$$

$$O.P.D.\ \Delta = 2\mu t \text{ .... in liquid film}$$

This is not true or total optical path difference. For true optical path difference it is necessary to consider phase reversal of π radians (path difference of $\lambda/2$) at point D.

Thus,

$$O.P.D.\ \Delta = 2\mu t - \frac{\lambda}{2}$$

CONDITION FOR BRIGHT RINGS

If optical path difference between the two rays is equal to an integral multiple of wavelength λ of incident light, then the two rays will interfere constructively,

$$\Delta = n\lambda$$

$$2\mu t - \frac{\lambda}{2} = n\lambda$$

$$2\mu t = (2n+1)\lambda/2\,, \quad \text{where } = 0, 1, 2, 3, \ldots \tag{3.43}$$

*In case of Newton's rings in reflected light with an air film, n cannot have a value of zero for bright rings. If we take $n = 0$, Eqn. (3.43) does not produce any mathematically illogical result but that is not allowed. Therefore condition given by Eqn. (3.43) can be written as

$$2\mu t = (2n-1)\lambda/2 \qquad \text{where n} = 1, 2, 3, \ldots$$

CONDITION FOR DARK RINGS

If the optical path difference between the two rays is odd integral number of half wavelength (λ/2) then the two rays will interfere destructively.

$$\Delta = (2n+1)\ \lambda/2$$

$$2\mu t - \frac{\lambda}{2} = (2n+1)\frac{\lambda}{2}$$

$$2\mu t = n\lambda \qquad n = 0, 1, 2, 3, \ \ldots \tag{3.44}$$

3.10.1 DETERMINATION OF DIAMETER OF BRIGHT AND DARK RINGS

The thickness t of the air film can be calculated in terms of the radii of the rings and radius of curvature of plano-convex lens as follows.

In the following fig. (3.15) a curved surface of the plano-convex lens is completed into a circle of radius R and C is the center of the circle.

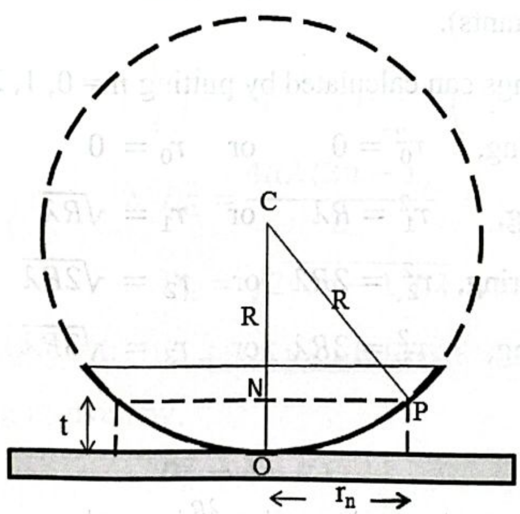


Fig: 3.15

Let O be the point of contact and let the n^{th} bright or dark ring of radius r_n is located at point P where the film thickness is t.

From right angle ΔCNP,

$$CP^2 = CN^2 + NP^2$$

$$CP^2 = (CO - ON)^2 + NP^2$$

$$R^2 = (R - t)^2 + NP^2$$

$$R^2 = (R - t)^2 + r_n^2$$

$$R^2 = R^2 - 2Rt + t^2 + r_n^2$$

Since the radius of curvature of lens R is very large compared to thickness t of the air film i.e. $2Rt \gg t^2$, therefore t^2 can be neglected.

$$2Rt = r_n^2$$

$$\therefore t = \frac{r_n^2}{2R} \tag{3.45}$$

We have condition for darkness in reflected light,

$$2\mu t = n\lambda \qquad n = 0, 1, 2, 3, \ldots\ldots. \tag{3.46}$$

Substituting $t = \frac{r_n^2}{2R}$ and $\mu_{air} = 1$, above condition becomes

$$\frac{2r_n^2}{2R} = n\lambda$$

$$r_n^2 = nR\lambda$$

$$\therefore r_n = \sqrt{nR\lambda} \tag{3.47}$$

$$\therefore r_n \propto \sqrt{n} \tag{3.48}$$

Eqn. (3.48) states that the radius of n^{th} dark ring is proportional to the square root of natural numbers ($\because$ R and λ are constants).

The radii of different dark rings can calculated by putting $n = 0, 1, 2, 3$

$$\left.\begin{array}{llll}
\text{For zero}^{th}\text{ order dark ring,} & r_0^2 = 0 & \text{or} & r_0 = 0 \\
\text{For first order dark ring,} & r_1^2 = R\lambda & \text{or} & r_1 = \sqrt{R\lambda} \\
\text{For second order dark ring,} & r_2^2 = 2R\lambda & \text{or} & r_2 = \sqrt{2R\lambda} \\
\text{For third order dark ring,} & r_3^2 = 3R\lambda & \text{or} & r_3 = \sqrt{3R\lambda}
\end{array}\right\}$$

and so on ...

Similarly, for bright rings,

$$r_n^2 = (2n - 1)\frac{\lambda R}{2} \tag{3.49}$$

$$\therefore r_n = \sqrt{\frac{(2n-1)R\lambda}{2}} \quad (3.50)$$

$$\therefore r_n \propto \sqrt{(2n-1)} \quad (3.51)$$

Thus the radius of n^{th} bright ring is proportional to square root of odd natural number ($\because$ 2, R and λ are constants).

The values of radii of different bright rings can calculated by putting n = 1, 2, 3,……

$$\left.\begin{array}{llll} \text{For first order bright ring,} & r_1^2 = \frac{3R\lambda}{2} & \text{or} & r_1 = \sqrt{\frac{3R\lambda}{2}} \\ \text{For second order bright ring,} & r_2^2 = \frac{5}{2}R\lambda & \text{or} & r_2 = \sqrt{\frac{5R\lambda}{2}} \\ \text{For third order bright ring,} & r_3^2 = \frac{7}{2}R\lambda & \text{or} & r_3 = \sqrt{\frac{7R\lambda}{2}} \end{array}\right\}$$

and so on…

For **dark rings**, we have

$$r_n^2 = nR\lambda$$

In terms of diameter,

$$\left(\frac{D_n}{2}\right)^2 = nR\lambda \quad \left(\because r_n = \frac{D_n}{2}\right)$$

$$\therefore D_n^2 = 4nR\lambda$$

$$\therefore D_n = \sqrt{4nR\lambda} \quad \text{where } n = 0, 1,2,3, \ldots. \quad (3.52)$$

For **bright rings**, we have

$$r_n^2 = (2n-1)\frac{\lambda R}{2}$$

In terms of diameter,

$$\left(\frac{D_n}{2}\right)^2 = (2n-1)\frac{\lambda R}{2} \quad \left(\because r_n = \frac{D_n}{2}\right)$$

$$\therefore D_n^2 = \frac{4R\lambda\,(2n-1)}{2}$$

$$\therefore D_n = \sqrt{2(2n-1)R\lambda} \quad \text{where } n = 1,2,3,\ldots \quad (3.53)$$

SPACING BETWEEN TWO CONSECUTIVE DARK OR BRIGHT RINGS

The diameter of n^{th} dark ring is given by,

$$D_n^2 = 4\,n\,R\lambda$$

$$D_n = 2\sqrt{n\,R\lambda} \quad (3.54)$$

The diameter of dark rings can be calculated by using $n = 0, 1,2,3,$ in equation (3.54).

For zeroth order dark ring, $D_0 = 0$

For first order dark ring, $D_1 = 2\sqrt{1\,R\lambda}$

For second order dark ring, $D_2 = 2\sqrt{2\,R\lambda}$

For third order dark ring, $D_3 = 2\sqrt{3\,R\lambda}$

For fourth order dark ring, $D_4 = 2\sqrt{4\,R\lambda}$

For fourteenth order dark ring, $D_{14} = 2\sqrt{14\,R\lambda}$

For fifteenth order dark ring, $D_{15} = 2\sqrt{15\,R\lambda}$

In Newton's rings experiment the separation between two consecutive dark (also bright) rings goes on decreasing as the order of the rings increases. This can be elucidated as follows.

$$D_2 - D_1 = 0.41,\ D_3 - D_2 = 0.32,\ D_4 - D_3 = 0.27 \ldots\ldots\ldots D_{15} - D_{14} = 0.13$$

From this it is clear that

$$(D_2 - D_1) > (D_3 - D_2) > (D_4 - D_3) > \ldots\ldots\ldots\ldots\ldots > D_{15} - D_{14}$$

Thus in Newton's ring pattern, the rings are seen coming closer to each other with increase in order of the ring.

3.10.2 Determination of Wavelength of Light/Radius of Curvature of Lens

Newton's ring pattern viewed in reflected light should have a dark spot at the center because the thickness of the air film at the point of contact is zero. However, if either the plane glass plate or the convex surface of the plano-convex lens contains dust particles or scratches then the center of the pattern may not be perfectly dark. Therefore the lens and the glass plate must be clean and scratch free. The center of the cross-wire of the eyepiece of the microscope is adjusted at the central dark spot of the pattern.

Then by counting the number of rings the microscope is moved to the right side of the central dark spot and the centre of cross wire is adjusted on tangential position of say 14th dark ring.

The position of microscope is noted. Then throughout the experiment the microscope is moved in opposite direction so that the position of microscope is noted for 12th, 10th, 8th, 6th, 4th and 2nd dark rings on right hand side of the center and then to the left hand side for 2nd, 4th etc. upto 14th dark ring as shown in Fig. (3.16).

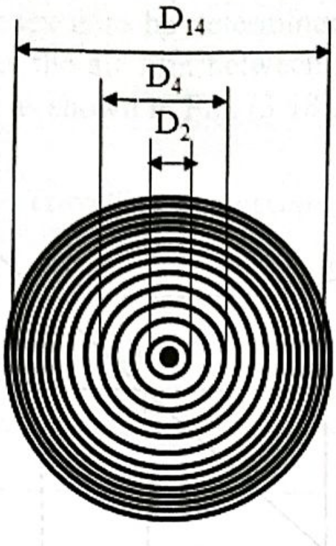


Fig: 3.16

We have the diameter of the n^{th} dark ring in reflected light as,

$$D_n^2 = 4\, n\, R\lambda \tag{3.55}$$

Diameter of $(n + p)^{th}$ dark ring is given by,

$$D_{n+p}^2 = 4\,(n + p)\, R\lambda \tag{3.56}$$

Subtracting equation (3.55) from equation (3.56) we get,

$$D_{n+p}^2 - D_n^2 = 4pR\lambda$$

$$\therefore\ \lambda = \frac{D_{n+p}^2 - D_n^2}{4pR} \tag{3.57}$$

If the wavelength λ of the monochromatic light used is known, then the radius of curvature R of plano-convex lens can be determined by the following equation,

$$R = \frac{D_{n+p}^2 - D_n^2}{4p\lambda} \tag{3.58}$$

GRAPHICAL METHOD

In practice, a graph is plotted between D_{n+p}^2 along y-axis and order of the ring n along x-axis, where n = 0, 1, 2, 3,… etc. The graph is a straight line as shown in Fig. (3.17) and slope of the graph is calculated which is given by,

$$\text{Slope} = \frac{D_{n+p}^2 - D_n^2}{p}$$

Thus equation (3.58) can be re-written as,

$$R = \frac{Slope}{4\lambda} \tag{3.59}$$

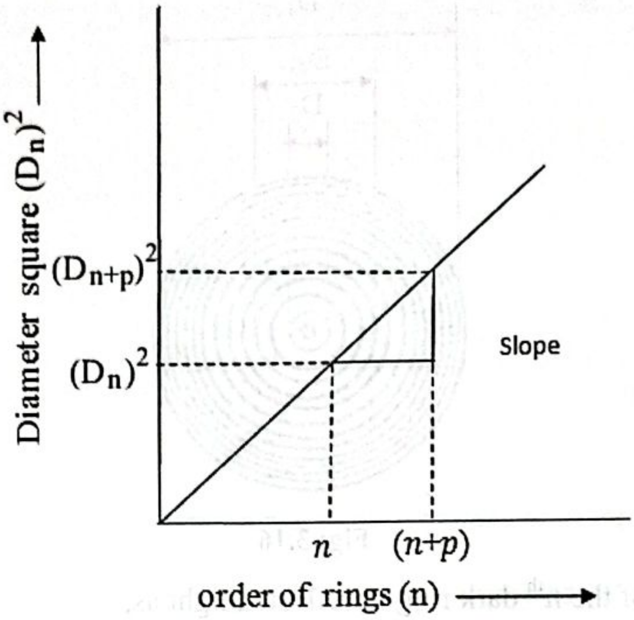


Fig: 3.17

DETERMINATION OF *R* USING SPHEROMETER

The radius of curvature of the plano-convex lens used in the Newton's rings experiment for measurement of wavelength of monochromatic light can be determined with the help of a spherometer using the formula,

$$R = \frac{l^2}{6h} + \frac{h}{2}$$

where, l is the distance between two legs of a spherometer, and h is the difference between the spherometer readings when it is placed on the curved surface of the lens and on a plane surface.

3.10.3 DETERMINATION OF REFRACTIVE INDEX OF LIQUID USING NEWTON'S RINGS

The experimental set-up consisting of a plano-convex lens placed on a plane glass plate is placed in a container. First the experiment is performed with an air film between the glass plate and the convex surface of the lens. The diameter of n^{th} and $(n + p)^{th}$ dark rings for air film are measured with help of traveling microscope, given by equations (3.60) and (3.61).

For air film,

$$(D_n^2)_{air} = 4\,n\,R\lambda \qquad (3.60)$$

$$\left(D_{n+p}^2\right)_{air} = 4\,(n+p)\,R\lambda \qquad (3.61)$$

Subtracting Eqn. (3.60) from Eqn. (3.61)

$$\left(D_{n+p}^2\right)_{air} - (D_n^2)_{air} = 4\,pR\lambda$$

(3.62)

Now the liquid whose refractive index is to be determined is poured slowly in the container without disturbing the system. Thus the air film between lens and glass plate is replaced by the liquid film of refractive index μ as shown in Fig. (3.18).

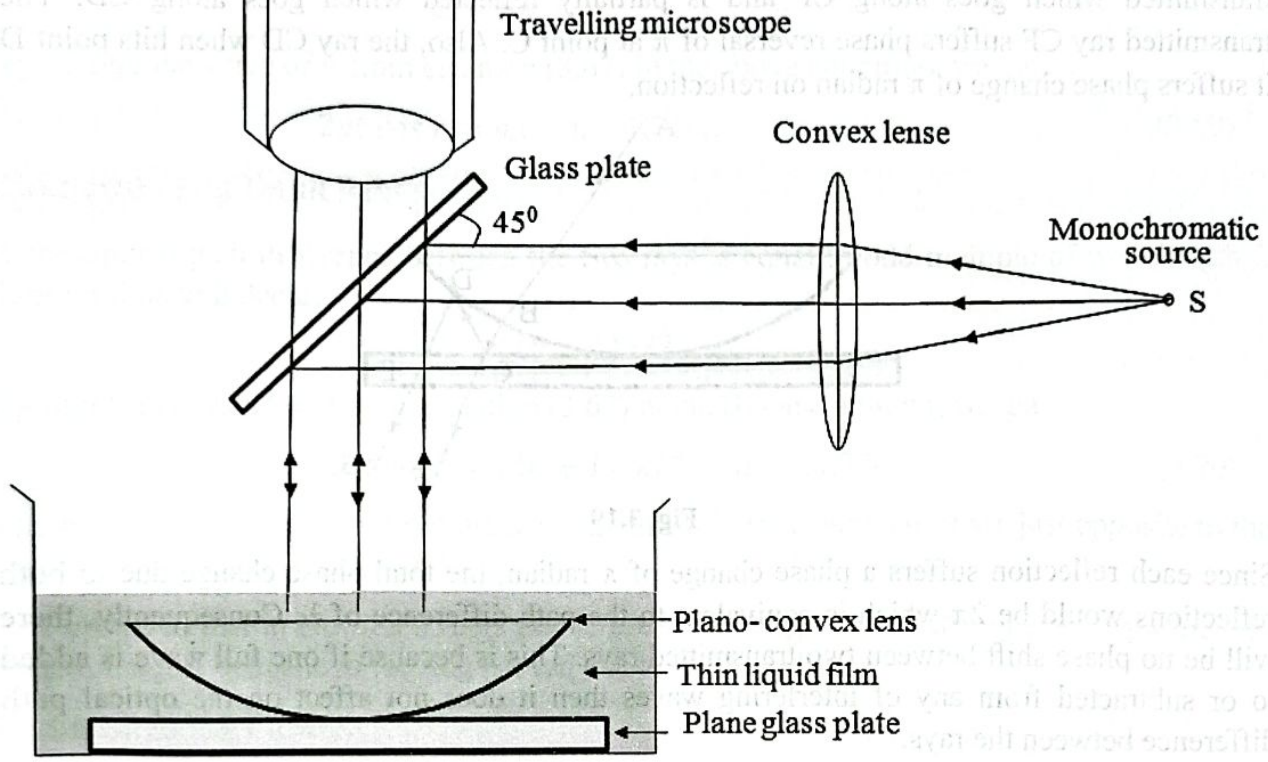


Fig: 3.18

The diameters of n^{th} and $(n+p)^{th}$ dark rings are measured for liquid film.

For liquid film,

$$(D_n'^2)_{liquid} = \frac{4\,n\,R\lambda}{\mu} \tag{3.63}$$

$$(D_{n+p}'^2)_{liquid} = \frac{4\,(n+p)\,R\lambda}{\mu} \tag{3.64}$$

Subtracting Eqn. (3.63) from Eqn. (3.64)

$$(D_{n+p}'^2 - D_n'^2)_{liquid} = \frac{4\,pR\lambda}{\mu}$$

$$\therefore \mu = \frac{4\,pR\lambda}{(D_{n+p}'^2 - D_n'^2)_{liquid}} \tag{3.65}$$

Putting Eqn. (3.62) in Eqn. (3.65) we get,

$$\mu = \frac{(D_{n+p}^2 - D_n^2)_{air}}{(D_{n+p}'^2 - D_n'^2)_{liquid}} \tag{3.66}$$

3.10.4 Newton's Rings in Transmitted Light

Newton's rings can also be observed in transmitted light. Fig. (3.19) shows that the rays CF and CDEG are the transmitted rays. From the figure it is clear that the ray BC is partially transmitted which goes along CF and is partially reflected which goes along CD. The transmitted ray CF suffers phase reversal of π at point C. Also, the ray CD when hits point D it suffers phase change of π radian on reflection.

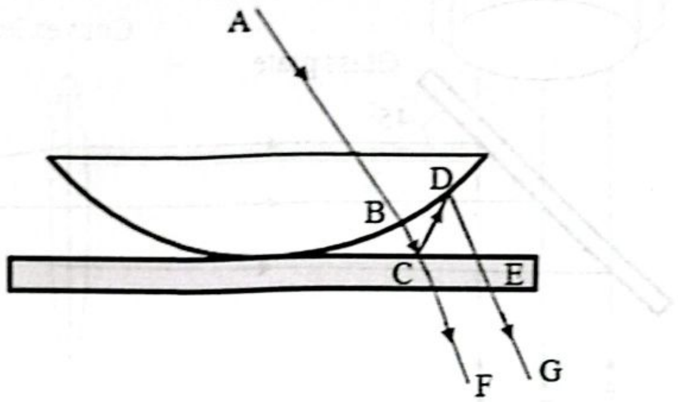


Fig. 3.19

Since each reflection suffers a phase change of π radian, the total phase change due to both reflections would be 2π which is equivalent to the path difference of λ. Consequently, there will be no phase shift between two transmitted rays. This is because if one full wave is added to or subtracted from any of interfering waves then it does not affect on the optical path difference between the rays.

The optical path difference between two transmitted rays CF and CDEG is therefore given by,

$$\Delta = 2\mu t \cos r \tag{3.67}$$

For normal incidence, $\cos r = 1$, and for air film $\mu_{air} = 1$.

So the optical path difference becomes,

$$\Delta = 2t \tag{3.68}$$

The path difference between the two transmitted waves at point of contact of lens and glass plate ($t = 0$), becomes equal to zero (i.e. $\Delta = 0$), which is the condition for maximum intensity. Hence the two transmitted waves interfere constructively. Therefore ***the centre of the ring system in transmitted light is bright*** as shown in Fig. (3.20).

Fig: 3.20 Newton's rings by transmitted light (centre of the system is bright)

CONDITION FOR BRIGHT RINGS

If the optical path difference between two rays is equal to an integral multiple of wavelength λ then maxima will occur.

$$\Delta = n\lambda$$

By putting the value of Δ from equation (3.67) in the above condition, we get,

$$2\mu t \cos r = n\lambda \quad n = 1,2,3,\ldots \qquad .(3.69)$$

CONDITION FOR DARK RINGS

If the optical path difference between the two rays is equal to odd multiple of wavelength λ then minima will occur.

$$\Delta = (2n+1)\lambda/2$$

By putting the value of Δ from equation (3.67) in the above condition, we get

$$2\mu t \cos r = (2n+1)\ \lambda/2 \quad n = 0,1,2,3,\ldots. \qquad (3.70)$$

The conditions obtained for brightness and darkness in transmitted rays are just opposite to the conditions obtained in reflected light.

It is observed that in Newton's rings due to transmitted light ***the centre of the ring system is bright***, i.e. just opposite to the ring pattern due to reflected light.

RADII OF BRIGHT RINGS

The condition for brightness in transmitted light is $2\mu t \cos r = n\lambda$. For normal incidence, $\cos r = 1$, and for air film, $\mu_{air} = 1$.

Substituting, $= \frac{r_n^2}{2R}$, we get, $r_n^2 = n\lambda R$ (3.71)

$$\therefore r_n = \sqrt{nR\lambda} \qquad (3.72)$$

$$\therefore r_n \propto \sqrt{n} \qquad (\because R \text{ and } \lambda \text{ are constants}) \qquad (3.73)$$

Thus the radii of n^{th} order bright rings in transmitted system are proportional to square root of natural numbers.

DIAMETER OF BRIGHT RINGS

The diameter of n^{th} order bright ring is given by,

$$D_n^2 = 4\,nR\lambda \qquad (\because r_n = \frac{D_n}{2}) \qquad (3.74)$$

$$\therefore D_n = 2\sqrt{nR\lambda} \qquad (3.75)$$

RADII OF DARK RINGS

The condition for dark ring in transmitted light is $2\mu t\cos r = (2n+1)\frac{\lambda}{2}$. For normal incidence, $\cos r = 1$, and for air film, $\mu_{air} = 1$.

Substituting, $= \frac{r_n^2}{2R}$, we get, $r_n^2 = \frac{(2n+1)R\lambda}{2}$ (3.76)

$$\therefore r_n = \sqrt{\frac{(2n+1)R\lambda}{2}} \quad (3.77)$$

$$\therefore r_n \propto \sqrt{(2n+1)} \quad (\because 2, R \text{ and } \lambda \text{ are constants}) \quad (3.78)$$

Thus the radius of n^{th} order dark rings in transmitted system is proportional to square root of odd natural numbers.

DIAMETER OF DARK RINGS

The diameter of n^{th} order dark ring is given by,

$$D_n^2 = \frac{4\,(2n+1)R\lambda}{2} \quad (\therefore r_n = \frac{D_n}{2}) \quad (3.79)$$

$$\therefore D_n = 2\sqrt{\frac{(2n+1)R\lambda}{2}} \quad (3.80)$$

In Newton's rings due to transmitted light the diameter of bright rings are proportional to the square root of natural numbers whereas the diameter of dark rings are proportional to the square root of odd natural number.

Fig: 3.21(a) Newton's ring by reflected light (centre of the system is dark)

Fig: 3.21(b) Newton's ring by transmitted light (centre of the system is bright)

It has been observed that the Newton's rings due to transmitted light are complementary to that seen in reflected light. However the Newton's rings seen in transmitted light are much poorer in contrast, because the transmitted rays emerge with lower intensity in comparison with the reflected rays.

3.10.5 NEWTON'S RINGS IN WHITE LIGHT

When a monochromatic source of light is used in the Newton's rings experiment, with an air film, the interference pattern consists of alternate dark and bright concentric rings with a central dark spot. We know that the diameter of the dark ring is proportional to wavelength of the light.

$$D_n^2 = 4\,nR\lambda$$

If instead of monochromatic light, white light is incident on the air film, coloured rings of different diameter would be seen. This is because the ring diameter depends upon the wavelength of light used. Due to overlapping of rings of different colours, only first few rings near the point of contact would be clearly visible while other rings cannot be viewed clearly.

3.10.6 NEWTON'S RINGS WITH BRIGHT CENTRE DUE TO REFLECTED LIGHT

Newton's rings are obtained by reflected light with a dark spot at the centre of the fringe pattern when there is an air film between the plano-convex lens and the plane glass plate. The thickness of air film is zero at the point of contact of the convex surface of the lens and the plane glass plate, but the two reflected rays suffers phase change of π radians which is equivalent to path difference of $\lambda/2$.

Let us consider that an air film is replaced by a transparent liquid of refractive index 'μ'. The refractive index of the material of the plano-convex lens is μ_1 and that of the plane glass plate is μ_2 such that $\mu_1 < \mu < \mu_2$. This is possible if a few drops of sasafaras oil are introduced between a plano-convex lens of crown glass and the plate of flint glass.

The ray BC is reflected from a boundary between an optically rarer and denser medium so it suffers phase change of π radian. Further the ray BD undergoes reflection from a boundary between an optically rarer and denser medium so it suffers phase change of π radian.

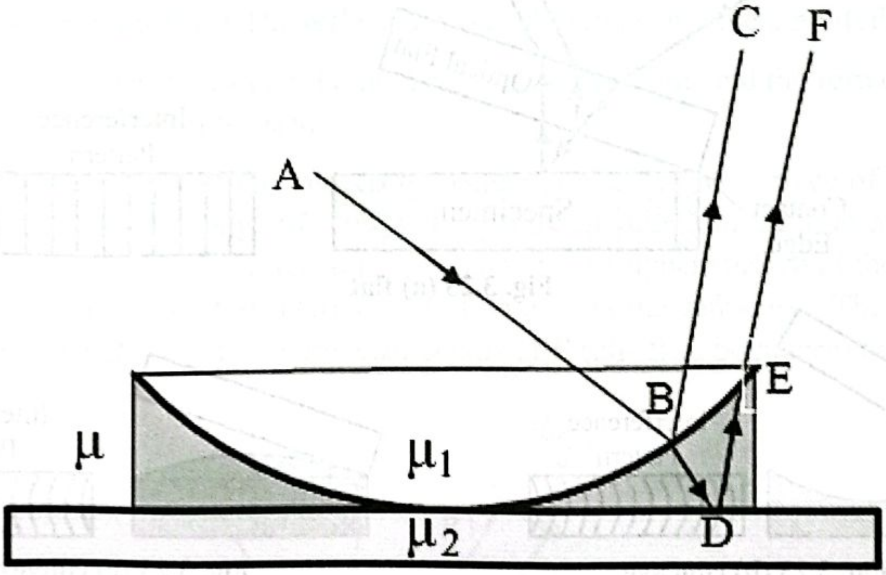


Fig. 3.22

Therefore the effective optical path difference between the reflected rays is given by,

$$\Delta = 2t \cos r$$

For normal incidence $\cos r = 1$

$$\Delta = 2\mu t$$

The thickness of air film is zero at the point of contact of lens and glass plate. The path difference between the reflected ray $\Delta = 0$, which is the condition for maximum intensity. The reflected rays interfere constructively and hence maximum intensity is observed at the centre of rings system i.e. ***the centre of system will appear bright.***

3.11 APPLICATIONS OF INTERFERENCE

Phenomenon of interference is used in variety of applications. Following are some of the applications of practical importance.

(a) Testing of machine components for flatness: The metallic discs or plates used as machine parts must have their surfaces highly finished. The flatness of these components can be tested using phenomenon of interference of light. An air wedge is formed by keeping an optical flat on the specimen surface at an angle and it is illuminated with monochromatic light.

If the specimen surface is optically plane then the fringes would be straight and of equal thickness. The fringes curved towards the contact edge indicate that the surface is concave and fringes curved away means that the surface is convex. The surface of the specimen is polished so that the air wedge produces interference pattern consisting of equidistant parallel fringes.

Fig. (3.23) shows three different fringe patterns.

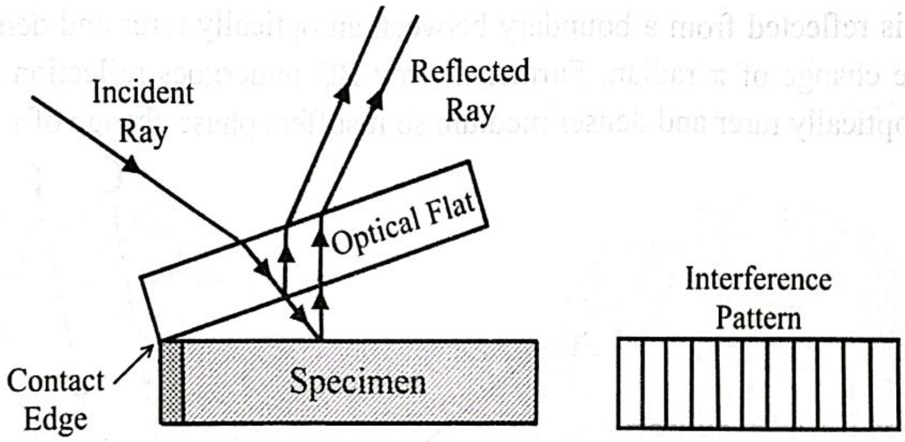


Fig. 3.23 (a) flat

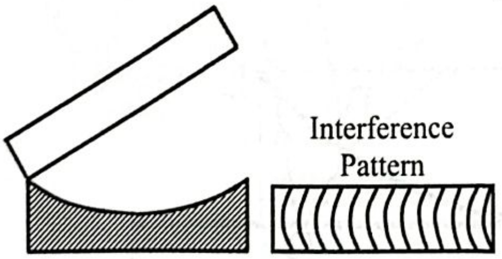


Fig. 3.23 (b) concave

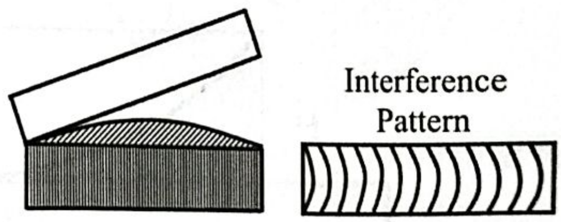


Fig. 3.23 (c) convex

(b) Testing of lens surface: The optical components such as lenses used in sophisticated instruments require testing for their optical flatness. Newton's rings experiment can be performed by placing a lens under test on an optically flat glass plate and the rings pattern can be observed. A circular fringe pattern is an indication of properly ground lens. Any variation in the fringe pattern suggests that the lens needs to be polished. The phenomenon of interference of light helps test finishing of lenses to the precision of less than a wavelength of light.

Fig. 3.24 (a) & (b) shows Newton's rings pattern in case of perfectly and roughly ground lens respectively.

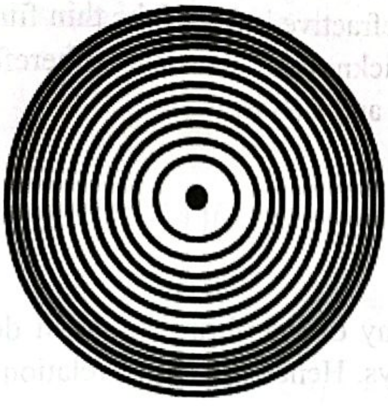


Fig: 3.24 (a)

Fig: 3.24 (b)

(c) Anti-reflection film: The lenses used for cameras and binoculars are coated with a thin transparent film to minimize the reflection of light incident on the lens surface. Such deposition of thin film on optical surfaces is known as antireflection coating. Reduction of reflection means less loss of incident light. This improves the quality of the image.

For a film to act as antireflection coating following two conditions must be satisfied.

1. **Phase condition:** The light waves reflected from the top and bottom surfaces of the deposited film must be out phase by 180° (equivalent to π radians).
2. **Amplitude condition:** The reflected waves should have same amplitude.

The above conditions determine the thickness of the AR film, and the refractive index of the material of the film to be used.

1. **Phase condition:** Let the ray AB be incident on the upper surface of the film. It is partially reflected along BR_1 and rest of it is transmitted in the film along BC. The ray BC is internally reflected at point C and hits the upper surface of the film at point D and undergoes partial internal reflection and partial refraction. The refracted ray goes along DR_2. The phase reversal occurs at B (air- film boundary) and at C (film - glass boundary).

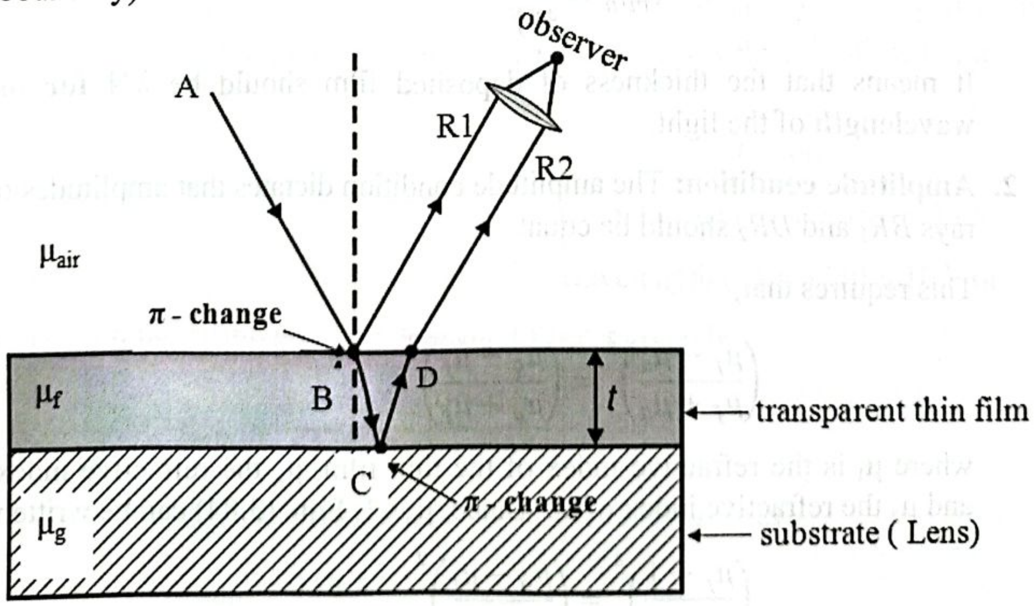


Fig: 3.25

Let μ_a is the refractive index of the air, μ_f is the refractive index of the thin film, μ_g is the refractive index of the substrate, t is the thickness of thin film. Therefore net optical difference between the reflected rays BR_1 and DR_2 is,

$$\Delta = 2\mu_f t\cos r - \frac{\lambda}{2} - \frac{\lambda}{2}$$

$$\Delta = 2\mu_f t\cos r - \lambda$$

If one full wave is added to or subtracted to any of interfering waves it does not affect the optical path difference between the rays. Hence the above relation can be written as

$$\Delta = 2\mu_f t\cos r \tag{3.81}$$

The reflected waves BR_1 and DR_2 interfere destructively if the net optical path difference satisfies the following condition.

$$\Delta = (2n+1)\frac{\lambda}{2}$$

By putting $\Delta = 2\mu_f t\cos r$ in above condition we get,

$$2\mu_f t\cos r = (2n+1)\frac{\lambda}{2}$$

For normal incidence, $\cos r = 1$

$$2\mu_f t = (2n+1)\frac{\lambda}{2} \tag{3.82}$$

The thickness of the deposited film will be minimum t_{min} when $= 0$

$$2u_f t_{min} = \frac{\lambda}{2}$$

$$t_{min} = \frac{\lambda}{4u_f} \tag{3.83}$$

It means that the thickness of deposited film should be $\lambda/4$ **for one particular wavelength** of the light.

2. **Amplitude condition:** The amplitude condition dictates that amplitudes of the reflected rays BR_1 and DR_2 should be equal.

   This requires that,

$$\left(\frac{\mu_f - \mu_a}{\mu_f + \mu_a}\right)^2 = \left(\frac{\mu_g - \mu_f}{\mu_g + \mu_f}\right)^2 \tag{3.84}$$

where μ_f is the refractive index of the thin film, μ_g the refractive index of the glass, and μ_a the refractive index of air. Since, $\mu_a = 1$, Eqn. (3.84) can be written as,

$$\left(\frac{\mu_f - 1}{}\right)^2 \quad \left(\frac{\mu_g - \mu_f}{}\right)^2$$

This can be simplified as,

$$\frac{(\mu_f - 1)}{(\mu_f + 1)} = \frac{(\mu_g - \mu_f)}{(\mu_g + \mu_f)}$$

$$(\mu_f - 1)(\mu_g + \mu_f) = (\mu_f + 1)(\mu_g - \mu_f)$$

$$\mu_f\mu_g + {\mu_f}^2 - \mu_g - \mu_f = \mu_f\mu_g - {\mu_f}^2 + \mu_g - \mu_f$$

$$2{\mu_f}^2 = 2\mu_g \qquad [since\ \mu_f \approx \mu_g]$$

$$\therefore \mu_f = \sqrt{\mu_g} \tag{3.85}$$

It is seen from Eqn. (3.85) that the refractive index of the thin transparent film deposited on the lens should be nearly equal to the square root of the refractive index of the lens.

In case of glass, $\mu_g = 1.5$

$$\therefore \mu_f = \sqrt{1.5} = 1.22$$

Magnesium fluoride, MgF_2 ($\mu = 1.38$) is preferred as antireflection coating because its refractive index is very near to the square root of the refractive index of glass. Anti-reflection coating on camera and telescope lenses are designed for a wavelength in the middle of the visible band giving reasonably good anti-reflection over the entire band.

The additional qualities of magnesium fluoride are that it is adhesive, scratch free, and durable. A transparent thin non-reflective film of silicon monoxide coated on the surface of a **solar cell** minimizes reflective losses.

Example 1: A transparent film of silicon mono-oxide having refractive index of 1.45 is coated on the surface of solar cell having refractive index of 3.5 to minimize reflection losses. What minimum thickness film of silicon monoxide should be deposited on the solar cell surface? (Wavelength of sun light taken as a mid wavelength of visible light = 5500 Å).

Solution: **Given**

refractive of the film (μ_f) = 1.45

wavelength (λ) = 5500×10^{-10} m

Formula: The minimum thickness of deposited film is given by,

$$t_{min} = \frac{\lambda}{4\mu_f} = \frac{5500 \times 10^{-10}}{4 \times 1.45}$$

$$t_{min} = 9.482 \times 10^{-8}\text{m} = 948.2 \times 10^{-10}\text{m}$$

$$t_{min} = 948\ \text{Å}$$

It implies that a transparent film of silicon mono-oxide having thickness of 948 Å should be deposited on the solar cell surface to minimize reflection losses.

(d) Highly Reflecting Film: A thin transparent film coated on a glass surface can act as highly reflecting film. Such coating is useful especially in sun glasses as well as in window glass of cars and buses.

A thin film of refractive index $\mu_a < \mu_f < \mu_g$ is deposited on a glass substrate. Ray AB incident on the upper surface of the thin film gets partially reflected and refracted at point B. The reflected ray goes along BR_1 and refracted ray along BC.

The refracted ray BC undergoes reflection at C and proceeds along CD. At point D ray CD refracts along DR_2. The phase reversal occurs at B (air- film boundary) and at C (film - glass boundary).

The path difference between rays BR_1 and $BCDR_2$ is

$$\Delta = 2\mu_f t \cos r - \frac{\lambda}{2} - \frac{\lambda}{2} \tag{3.86}$$

$$\therefore \ \Delta = 2\mu_f t \cos r - \lambda \tag{3.87}$$

If one full wave is added or subtracted from any of interfering waves it does not affect the optical path difference between the rays.

Hence the effective optical path difference becomes

$$\Delta = 2\mu_f t \cos r \tag{3.88}$$

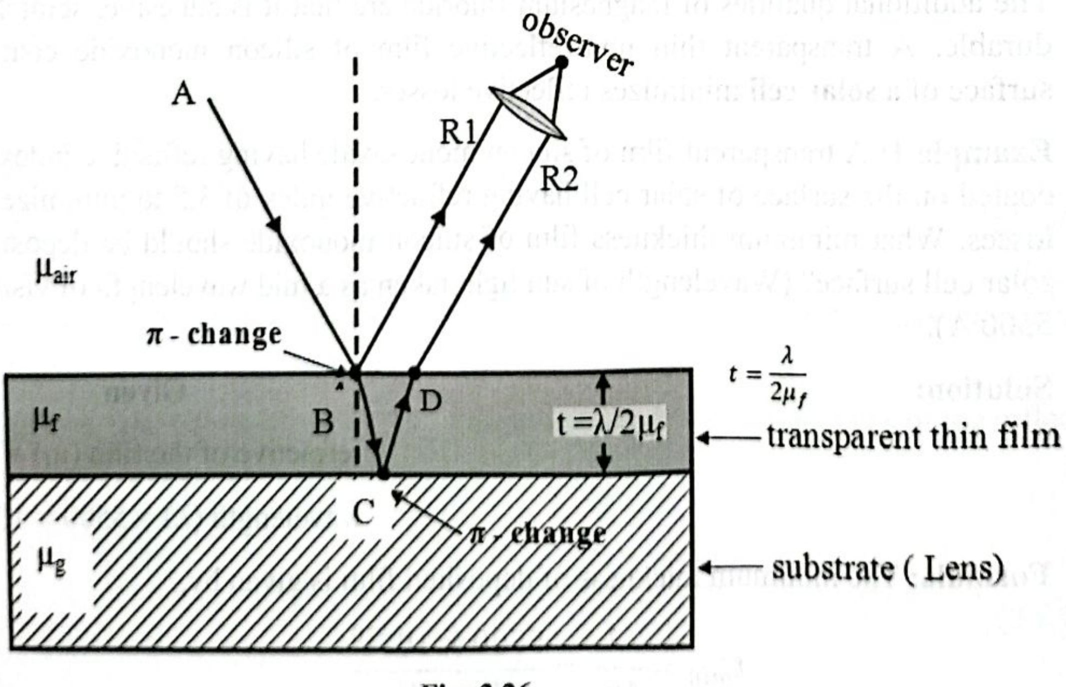


Fig: 3.26

The reflected rays BR_1 and DR_2 interfere constructively if the net optical path difference is equal to an integral multiple full wavelength.

$$\Delta = n\lambda$$

Putting $\Delta = 2\mu_f t\cos r$ in above equation we get,

$$2\mu_f t\cos r = n\lambda \tag{3.89}$$

For normal incidence, $r = 1$,

$$2\mu_f t = n\lambda \tag{3.90}$$

Thus if the thickness of the deposited film $= n\lambda/2\mu_f$, then on account of constructive interference the film acts as highly reflecting film.

Why are Newton's rings circular?

The locus of all points having the same thickness of air column is circular with its centre at the point of contact of plane glass plate and plano-convex lens. Light reflected from all these points will have same brightness (or darkness). Therefore Newton's rings are circular

SOLVED PROBLEMS

1. A parallel beam of light (λ = 5870 Å) is incident on a thin glass plate (μ=1.5), such that the angle of refraction into the plate is 60^0. Calculate the smallest thickness of the glass plate which will appear dark by reflection.

Solution: **Given**

Wavelength of the light (λ) = 5870 Å = 5870×10^{-10} m

Refractive index of glass plate (μ) = 1.5

Angle of refraction (r) = 60^0

Formula: The condition for n^{th} order darkness (reflected system)

$$2\mu t\cos r = n\lambda \qquad \text{where } n = 1, 2, 3\ldots\ldots \text{ etc.}$$

For minimum thickness, $t = t_{min}$, $n = 1$

Above condition becomes,

$$2\mu t_{min} \cos r = 1 \times \lambda$$

$$t_{min} = \frac{\lambda}{2\mu \cos r} = \frac{5870 \times 10^{-10}}{2 \times 1.5 \times \cos 60^0}$$

$$= 3.913 \times 10^{-7} \text{ m} = 3913 \times 10^{-10} \text{ m}$$

The smallest thickness of the glass plate (t_{min}) = 3913 Å

2. A soap of refractive index 1.33 is illuminated with light of different wavelengths at an angle of 45^0. Calculate the smallest thickness of the film which will appear dark by reflection. Given wavelength of light used λ = 5890 Å.

Solution: **Given**

Refractive index (μ) = 1.33

Angle of incidence (i) = 45°.

Wavelength of light (λ) = 5890 Å = 5890×10^{-10} m

Formula: Calculation of cos r

We have Snell's law,

$$\mu = \frac{\sin i}{\sin r}$$

$$\sin r = \frac{\sin i}{\mu} = \frac{\sin 45}{1.33}$$

$$\sin r = 0.5316 \Rightarrow r = 32.11°$$

$$\therefore \cos r = 0.8470$$

Calculation of minimum thickness of film

Condition for darkness in the reflected light

$$2\mu t \cos r = n\lambda \qquad n = 1, 2, 3, \ldots \text{etc.}$$

For minimum thickness film, n = 1

$$t_{min} = \frac{n\lambda}{2\mu\cos r} = \frac{1 \times 5890 \times 10^{-10}}{2 \times 1.33 \times 0.8470}$$

$$= 2.614 \times 10^{-7} \text{ m} = 2614 \times 10^{-10} \text{ m}$$

The smallest thickness of the film (t_{min}) = 2614 Å

3. A soap film of 5×10^{-5} cm thick is viewed at an angle of 35^0 to the normal. Find the wavelength of the light in the visible spectrum which will be absent from the reflected light (μ = 1.33).

Solution:

Given

Angle of incident (i) = 35°

Thickness of soap film (t) = 5×10^{-5} cm

Refractive index of soap film (μ) = 1.33

Formula: Calculation of cos r

We have Snell's law, $\mu = \dfrac{\sin i}{\sin r}$

$$\sin r = \frac{\sin i}{\mu} = \frac{\sin 35}{1.33}$$

$$\sin r = 0.4312 \Rightarrow r = 25.54°$$

$$\therefore \cos r = 0.9022$$

Calculation of wavelength of various orders

The wavelength of visible light (4000 to 7800 Å) which will be absent from the reflected light can be calculated using condition of destructive interference of light.

Condition of darkness for reflected rays is given by,

$$2\mu t \cos r = n\lambda \quad n = 1, 2, 3, \ldots \text{ etc.}$$

$$\lambda = \frac{2\mu t \cos r}{n}$$

The value of wavelength of various orders can calculated by putting n = 1, 2, 3

For first order, n = 1

$$\lambda_1 = \frac{2 \times 1.33 \times 5 \times 10^{-5} \times 0.9022}{1}$$

$$\lambda_1 = 12 \times 10^{-5} \text{ cm} = 12000 \times 10^{-8} \text{ cm}$$

$\lambda_1 = 12000$ **Å (lies in the infrared region)**

For second order, n = 2

$$\lambda_2 = \frac{2 \times 1.33 \times 5 \times 10^{-5} \times 0.9022}{2}$$

$$\lambda_2 = 6 \times 10^{-5} \text{ cm} = 6000 \times 10^{-8} \text{ cm}$$

$\lambda_2 = 6000$ **Å (lies in the visible region)**

For third order, n = 3

$$\lambda_3 = \frac{2 \times 1.33 \times 5 \times 10^{-5} \times 0.9022}{3}$$

$$\lambda_3 = 4 \times 10^{-5} \text{ cm} = 4000 \times 10^{-8} \text{ cm}$$

$\lambda_3 = 4000$ **Å (lies in the visible region)**

For fourth order, n = 4

$$\lambda_4 = \frac{2 \times 1.33 \times 5 \times 10^{-5} \times 0.9022}{4}$$

$$\lambda_4 = 3 \times 10^{-5} \text{ cm} = 3000 \times 10^{-8} \text{ cm}$$

$\lambda_4 = 3000$ **Å (lies in the ultra violet region)**

Hence the wavelength $\lambda_3 = 6000$ Å and $\lambda_4 = 4000$ Å are absent in the reflected light.

4. White light is incident on a soap film at an angle $\sin^{-1}(4/5)$ and the reflected light is observed with a spectroscope it is found that two consecutive dark bands corresponds to wavelengths $\lambda_1 = 6100$ Å and $\lambda_2 = 6000$ Å. Calculate the thickness of soap film. Given that the refractive index of the soap film (μ) = 4/3.

Solution:

Given

Refractive index of soap film (μ) = 4/3

Angle of incidence (i) = $\sin^{-1}(4/5)$

Wavelength (λ_1) = 6100 Å = 6100×10^{-10} m

Wavelength (λ_2) = 6000 Å = 6000×10^{-10} m

Formula: Calculation of order of dark band

The condition for darkness in the reflected system is given by,

$$2\mu t\cos r = n\lambda \tag{1}$$

If n^{th} dark band corresponds to wavelength (λ_1) and $(n+1)^{th}$ dark band corresponds to wavelength (λ_2) then equation (1) can be written as

$$2\mu t\cos r = n\lambda_1 \tag{2}$$

$$2\mu t\cos r = (n+1)\lambda_2 \tag{3}$$

From equation (2) and (3) $\quad n\lambda_1 = (n+1)\lambda_2$

$$\therefore n = \frac{\lambda_2}{\lambda_1 - \lambda_2} = \frac{6000 \times 10^{-10}}{(6100 - 6000) \times 10^{-10}}$$

Order of the band (n) = 60

Calculation of cosr

We have, Snell's law, $\quad \mu = \dfrac{\sin i}{\sin r}$

$$\sin r = \frac{4/5}{4/3} = \frac{4}{5} \times \frac{3}{4} = \frac{3}{5}$$

$$r = \sin^{-1}\left(\frac{3}{5}\right) = 36.86°$$

$$\cos r = 0.8001$$

Calculation of thickness of soap film

Equation (2) becomes, $2\mu t\cos r = n\lambda_1$

$$t = \frac{n\lambda_1}{2\mu\cos r} = \frac{60 \times 6100 \times 10^{-10}}{2 \times \frac{4}{3} \times 0.8001}$$

Thickness of soap film (t) = 1.71×10^{-5} m

5. A wedge shaped air film is illuminated by light of wavelength 4650 Å. The angle of wedge is 40 seconds. Calculate the separation between two consecutive fringes.

Solution: **Given**

Refractive index (μ_{air}) = 1

Wavelength of light (λ) = 4650 Å = 4650×10^{-10} m

Angle of wedge (α) = 40 seconds

$$\frac{40}{60 \times 60} \times \frac{\pi}{180} = 1.939 \times 10^{-4} \text{ radians}$$

$$\text{Fringe width } (\beta) = \frac{\lambda}{2\mu\alpha} = \frac{4650 \times 10^{-10}}{2 \times 1 \times 1.939 \times 10^{-4}}$$

Fringe width (β) = 1.199×10^{-3} meter

6. A wedge air film is enclosed between two glass plates in contact at one edge and separated by a wire of 0.06×10^{-3} m, diameter at a distance of 0.15 m from the edge. Calculate the fringe width. Given- the light wavelength 6000 A^0 is used.

Solution:

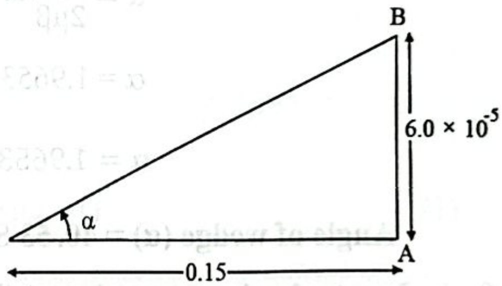
Given

Thickness of wire (t) = 0.06×10^{-3} m

Length of air film (l) = 0.15 m

Light of wavelength (λ) = 6000 Å = 6000×10^{-10} m

Formula: $\text{Fringe width } (\beta) = \frac{\lambda}{2\mu\alpha}$



$$\tan\alpha \cong \alpha = \frac{0.06 \times 10^{-3}}{0.15}$$

$$\beta = \frac{6000 \times 10^{-10} \times 0.15}{2 \times 1 \times 0.06 \times 10^{-3}}$$

$$= 7.5 \times 10^{-4} \text{ m } (\mu_{air} = 1)$$

$$\beta = 7.5 \times 10^{-4} \text{ m}$$

The fringe width (β) = 0.75×10^{-3} m = 0.75 mm

7. Interference fringes are produced with monochromatic light falling normally on a wedge shaped air film of cellophane whose refractive index is 1.40. The angle of wedge is 10 seconds of an arc and distance between successive fringes is 0.5 cm. Calculate wavelength of light used.

Solution:

Given

Wedge angle (α) = 10 second

Fringe width (β) = 0.5 cm

Refractive index (μ) = 1.40

$$10 \text{ seconds} = \frac{10 \times \pi}{60 \times 60 \times 180} = 3.848 \times 10^{-5} \text{rad}$$

Formula: $\text{Fringe width } (\beta) = \frac{\lambda}{2\mu\alpha}$

$$\lambda = 2\mu\alpha\beta$$

$$\lambda = 2 \times 1.40 \times 3.848 \times 10^{-5} \times 0.5$$

$$\lambda = 6787 \times 10^{-8} \text{ cm}$$

Wavelength of light (λ) = 6787 Å

8. Fringes of equal thickness are observed in a thin glass wedge of refractive index 1.5 the fringe spacing is 1mm and the wavelength of light is 5896 Å. Calculate angle of wedge in seconds of an arc.

Solution: **Given**

Refractive index (μ) = 1.5

Fringe width (β) = 1 mm = 1×10^{-3} m

Wavelength of light (λ) = 5896Å = 5896×10^{-10}

Formula: fringe width $(\beta) = \frac{\lambda}{2\mu\alpha}$

$$\alpha = \frac{\lambda}{2\mu\beta} = \frac{5896 \times 10^{-10}}{2 \times 1.5 \times 1 \times 10^{-3}}$$

$$\alpha = 1.9653 \times 10^{-4} \text{ radians}$$

$$\alpha = 1.9653 \times 10^{-4} \times \frac{180}{\pi} \times 60 \times 60 \text{ seconds}$$

Angle of wedge (α) = 40.53 Seconds

9. In Newton's rings experiment the diameter of fifth dark ring is 0.65 cm and that of fifteenth dark is ring is 0.95 cm. If wavelength of the source used is 6000 Å. Calculate the radius of curvature of a convex surface of the lens in contact with the glass plate.

Solution: **Given**

Diameter of 5th dark ring (D_5) = 0.65 cm

Diameter of 15th dark t ring (D_{15}) = 0.95 cm

Wavelength of the light (λ) = 6000A^0 = 6000×10^{-8} cm

Formula: we have, $\lambda = \frac{D_{n+p}^2 - D_n^2}{4pR}$

$$\therefore R = \frac{D_{n+p}^2 - D_n^2}{4p\lambda}$$

$$R = \frac{D_{15}^2 - D_5^2}{4p\lambda} = \frac{(0.95)^2 - (0.65)^2}{4 \times 10 \times 6000 \times 10^{-8}} = 200$$

The radius of curvature of a convex lens (R) = 200 cm

10. Newton's rings are obtained with reflected light of wavelength 5893 Å. The diameter of 10th dark ring is 5 mm. Now the space between the lens and the glass plate is filled with a liquid of refractive index 1.25. What is diameter of 10th dark ring now?

Solution: **Given**

Wavelength of light (λ) = 5893A^0 = 5893×10^{-8} cm

Diameter of 10th dark ring (D_{10}) = 5 mm = 0.5 cm

Order of dark ring (n) = 10

Refractive index of the liquid (μ) = 1.25

Formula: The diameter nth order dark ring is given by,

$$D_n^2 = \frac{4nR\lambda}{\mu}$$

For air film,

The diameter 10th dark order ring is given by,

$$D_n^2 = \frac{4nR\lambda}{\mu}$$

$$D_{10}^2 = 4nR\lambda \qquad (\mu_{air} = 1) \qquad (1)$$

For liquid film,

Let D'_{10} be diameter of 10th order dark ring & given by

$$D_{10}'^{\,2} = \frac{4 \times n\,R\,\lambda}{\mu} \qquad (2)$$

Divide equation (1) by (2)

$$\mu = \frac{D_{10}^2}{D_{10}'^{\,2}}$$

$$D_{10}'^{\,2} = \frac{D_{10}^2}{\mu} = \frac{(0.5)^2}{1.25}$$

$$D'_{10} = 0.4472 \text{ cm}$$

The diameter of 10th dark ring for liquid film D'_{10} = 0.4472 cm

11. In Newton's ring an experiment the diameter of 5th ring was 0.336 cm and that of 15th ring was 0.590 cm. Find radius of curvature of plano-convex lens, if the wavelength of 5890 A^0 is used.

Solution: **Given**

Diameter of 5th ring (D_5) = 0.336 cm

Diameter of 15th ring (D_{15}) = 0.590 cm

Wavelength of light (λ) = 5890 Å = 5890×10^{-8} cm

Formula: $\lambda = \dfrac{D_{n+p}^2 - D_n^2}{4pR}$

$$R = \frac{D_{n+p}^2 - D_n^2}{4p\lambda}$$

$$R = \frac{D_{15}^2 - D_5^2}{4p\lambda} = \frac{(0.590)^2 - (0.336)^2}{4 \times 10 \times 5890 \times 10^{-8}}$$

The radius of curvature of plano-convex lens (R) = 99.83 cm

12. Show with clear examples that separation between two consecutive similar rings in Newton's ring experiment goes on reducing as the series number of ring increases.

Solution: The diameter of n^{th} order dark ring is given by,

$$D_n^2 = 4\, n\, R\lambda$$

$$D_n = 2\sqrt{n\, R\lambda} \qquad (1)$$

The diameter of different order dark rings can calculated by putting $n = 0,1,2,3,..$ in equation (1)

For zeroth order dark ring, $D_0 = 0$

For first order dark ring, $D_1 = 2\sqrt{1\, R\lambda} = 2\sqrt{R\lambda}$

For second order dark ring, $D_2 = 2\sqrt{2\, R\lambda}$

For third order dark ring, $D_3 = 2\sqrt{3\, R\lambda}$

For fourth order dark ring, $D_4 = 2\sqrt{4\, R\lambda}$

For seventieth order dark ring, $D_{70} = 2\sqrt{70\, R\lambda}$

For seventy-one th order dark ring, $D_{71} = 2\sqrt{71\, R\lambda}$

In Newton's rings experiment the separation between two consecutive dark (also bright) rings goes on reducing as the series number of ring increases. This can be elucidated as follows.

$D_2 - D_1 = 0.41$, $D_3 - D_2 = 0.32$, $D_4 - D_3 = 0.27$ and so on… $D_{71} - D_{70} = 0.06$

From this it is clear that

$(D_2 - D_1) > (D_3 - D_2) > (D_4 - D_3) > \ldots\ldots\ldots\ldots > (D_{71} - D_{70})$

Thus in the Newton's rings pattern the rings are seen coming close with increase in the order of the ring.

13. Newton's ring arrangement with a source emitting two wavelengths $\lambda_1 = 6000$ Å and $\lambda_2 = 4500$ Å and it is found that nth dark ring due to wavelength λ_1 coincides with (n+1)th dark ring due to wavelength λ_2. Find the diameter of nth dark ring of λ_1 given that the radius of curvature of the convex lens (R) = 90 cm.

Solution: **Given**

Wavelength (λ_1) = 6000 Å = 6000 × 10^{-8} cm

Wavelength (λ_2) = 4500 Å = 4500 × 10^{-8} cm

Radius of curvature (R) = 90 cm

Formula: The diameter of nth order dark ring is given by,

$$D_n^2 = 4nR\lambda \tag{1}$$

The diameter of nth order dark ring corresponds to wavelength λ_1 is given by,

$$D_n^2 = 4nR\lambda_1 \tag{2}$$

The order (n+1)th order dark corresponds to wavelength λ_2 is given by,

$$D_{n+1}^2 = 4(n+1)R\lambda_2 \tag{3}$$

The diameter of nth order dark ring due to λ_1 coincides with (n+1)th order due to λ_2

$$D_n^2 = D_{n+1}^2$$

$$4nR\lambda_1 = 4(n+1)R\lambda_2$$

$$n\lambda_1 = (n+1)R\lambda_2$$

$$n\lambda_1 = n\lambda_2 + \lambda_2$$

$$n(\lambda_1 - \lambda_2) = \lambda_2$$

$$n = \frac{\lambda_2}{\lambda_1 - \lambda_2} = \frac{4500 \times 10^{-8}}{(6000 - 4500) \times 10^{-8}}$$

Order of the ring (n) = 3

Substitute (n = 3) in equation (2)

$$D_n^2 = 4nR\lambda_1$$

$$D_n^2 = 4 \times 3 \times 90 \times 6000 \times 10^{-8} = 0.0648$$

The diameter of nth dark ring (D_n) = 0.2545 cm.

14. Light containing two wavelengths, λ_1 and λ_2 falls normally on a convex lens of radius of curvature R resting on a glass plate. Now if nth dark ring due to λ_1 coincides with (n+1)th dark ring due to light of wavelength λ_2. Show that radius on nth dark ring is given by, $r_n = \sqrt{\frac{\lambda_1\lambda_2 R}{\lambda_1 - \lambda_2}}$.

Solution: The diameter of nth order dark ring is given by,

$$D_n^2 = 4nR\lambda \tag{1}$$

The diameter of n[th] order ring corresponds to wavelength λ_1 is given by,

$$D_n^2 = 4nR\lambda_1 \tag{2}$$

The diameter of (n+1)[th] order dark rings corresponds to wavelength λ_2 is given by,

$$D_{n+1}^2 = 4(n+1)R\lambda_2 \tag{3}$$

Diameter of n[th] dark ring due to wavelength λ_1 coincides with diameter of (n+1)[th] dark ring due to wavelength λ_2

$$D_n^2 = D_{n+1}^2$$

From equations (2) and (3), $4nR\lambda_1 = 4(n+1)R\lambda_2$

$$\therefore n = \frac{\lambda_2}{\lambda_1 - \lambda_2} \tag{4}$$

By using equation (4) in equation (2),

$$D_n^2 = 4nR\lambda_1$$

$$D_n^2 = 4\left(\frac{\lambda_2}{\lambda_1 - \lambda_2}\right)R\lambda_1$$

$$(2r_n)^2 = 4\left(\frac{\lambda_2}{\lambda_1 - \lambda_2}\right)R\lambda_1 \qquad (D_n = 2\,r_n)$$

$$4r_n^2 = 4\frac{\lambda_2\lambda_1 R}{\lambda_1 - \lambda_2}$$

$$r_n^2 = \frac{\lambda_1\lambda_2 R}{(\lambda_1 - \lambda_2)}$$

$$r_n = \sqrt{\frac{\lambda_1\lambda_2 R}{\lambda_1 - \lambda_2}}$$

15. Newton's ring arrangement is used with a source emitting two wavelengths λ_1 and λ_2. It is found that the n[th] dark ring due to λ_1 coincides with (n+1)[th] dark ring due to λ_2. Find the diameter of the n[th] dark ring for wavelength λ_1. Given λ_1 = 6000 Å, λ_2 = 5900 Å and radius of curvature of the lens is 90 cm.

Solution: **Given**

$\lambda_1 = 6000$ Å $= 6000 \times 10^{-8}$ cm,

$\lambda_2 = 5900$ Å $= 5900 \times 10^{-8}$ cm

$R = 90$ cm

Formula: We have, $D_n = 2\,r_n = 2 \times \sqrt{\dfrac{\lambda_1\lambda_2 R}{(\lambda_1-\lambda_2)}}$

$$= 2 \times \sqrt{\frac{6000 \times 10^{-8} \times 5900 \times 10^{-8} \times 90}{(6000 - 5900) \times 10^{-8}}}$$

$$= 0.128$$

The diameter of the n^th^ dark ring (D_n) = 0.128 cm

16. In Newton's rings experiment what will be the order of the dark ring which will have double diameter of that of 20^th^ dark ring given the wavelength of incident light is 5893 Å.

Solution:

Given

Dark ring (n) = 20

Wavelength (λ) = 5893 Å

Formula: The diameter n^th^ order dark ring is give by,

$$D_n^2 = 4nR\lambda \tag{1}$$

The diameter of 20^th^ order dark ring is given by,

$$D_{20}^2 = 4 \times 20\ R\lambda \tag{2}$$

Let n be the order of the dark ring whose diameter is twice that of the 20^th^ dark ring.

$$D_n = 2D_{20}$$

We have

$$D_n^2 = 4n\ R\lambda$$

$$(2D_{20})^2 = 4\ n\ R\ \lambda$$

$$4D_{20}^2 = 4\ n\ R\lambda \tag{3}$$

Dividing equation (2) by (3),

$$\frac{D_{20}^2}{4D_{20}^2} = \frac{4 \times 20 \times R \times \lambda}{4 \times n \times R \times \lambda}$$

The order of 40^th^ dark ring (n) = 80

17. In Newton's Ring experiment the n^th^ dark ring due to light of wavelength λ_1 coincides with (n+2)^th^ dark ring due to light of wavelength λ_2. Show that radius on n^th^ dark ring is given by, $r_n = \sqrt{\dfrac{2\lambda_1\lambda_2 R}{(\lambda_1-\lambda_2)}}$

Solution: The diameter of n^th^ order dark ring is given by,

$$D_n^2 = 4nR\lambda \tag{1}$$

The diameter of n^{th} order dark ring corresponds to wavelength λ_1 is given by,

$$D_n^2 = 4nR\lambda_1 \tag{2}$$

The order $(n+2)^{th}$ order dark corresponds to wavelength λ_2 is given by,

$$D_{n+2}^2 = 4(n+2)R\lambda_2 \tag{3}$$

The diameter of n th order dark ring due to λ_1 coincides with $(n+2)^{th}$ order due to λ_2

$$D_n^2 = D_{n+2}^2$$

$$4nR\lambda_1 = 4(n+2)R\lambda_2$$

$$n\lambda_1 = n\lambda_2 + 2\lambda_2$$

$$n(\lambda_1 - \lambda_2) = 2\lambda_2$$

$$n = \frac{2\lambda_2}{(\lambda_1 - \lambda_2)} \tag{4}$$

On putting value of n in equation (1)

$$D_n^2 = 4\frac{2\lambda_2}{(\lambda_1 - \lambda_1)}R\lambda_1$$

$$(2r_n)^2 = 4\frac{2\lambda_1\lambda_2 R}{(\lambda_1 - \lambda_2)} \qquad (\because D_n = 2\,r_n)$$

$$r_n^2 = \frac{2\lambda_1\lambda_2 R}{(\lambda_1 - \lambda_2)}$$

$$r_n = \sqrt{\frac{2\lambda_1\lambda_2 R}{(\lambda_1 - \lambda_2)}}$$

18. In a Newton's ring experiment the diameter of 5^{th} dark ring was 0.336 cm. Find the radius of curvature of the plano- convex lens, if the wavelength of light used is 5880 Å. Also find the radius of 15^{th} dark ring. (Given $\mu = 1$)

Solution: **Given**

Diameter of 15^{th} dark ring (D_{15}) = 0.336 cm

Wavelength of the light (λ) = 5880 Å = 5880×10^{-8} cm

Refractive index of medium (μ) =1

Formula: The diameter of n^{th} order dark ring is given by,

$$D_n^2 = 4nR\lambda$$

Calculation of the radius of curvature of the plano- convex lens

$$R = \frac{D_n^2}{4n\lambda} = \frac{(0.336)^2}{4 \times 3 \times 5880 \times 10^{-8}} = 160 \text{ cm}$$

The radius of curvature of the lens (R) = 160 cm

Calculation of the radius of of 15[th] dark ring

$$D_n^2 = 4nR\lambda$$

$$r_n^2 = nR\lambda \qquad \left(\because r_n = \frac{D_n}{2}\right)$$

$$r_{15}^2 = 15 \times 160 \times 5880 \times 10^{-8}$$

$$r_{15} = \sqrt{15 \times 160 \times 5880 \times 10^{-8}}$$

$$= 0.375$$

The radius of 15[th] dark ring (r_{15}) = 0.375 cm

19. In Newton ring experiment the diameter of 4[th] and 12[th] dark rings are 0.40 cm & 0.70 cm. Find diameter of 16[th] dark ring.

Solution:

Given

$D_4 = 0.40$ cm

$D_{12} = 0.70$ cm

Formula: $\lambda = \dfrac{D_{n+p}^2 - D_n^2}{4pR}$

$$D_{(n+p)}^2 - D_n^2 = 4pR\lambda \qquad (1)$$

$$D_{12}^2 - D_4^2 = 4 \times 8 \times R\lambda \qquad (2)$$

$$D_{16}^2 - D_4^2 = 4 \times 12 \times R\lambda \qquad (3)$$

Dividing equation (3) by (2) we get,

$$\frac{D_{16}^2 - D_4^2}{D_{12}^2 - D_4^2} = \frac{4 \times 12 \times R\lambda}{4 \times 8 \times R\lambda}$$

$$\frac{D_{12}^2 - D_4^2}{D_{16}^2 - D_4^2} = \frac{3}{2}$$

$$\frac{(0.70)^2 - (0.40)^2}{D_{16}^2 - (0.40)^2} = \frac{3}{2}$$

The diameter of 16[th] dark ring (D_{16}) = 0.61 cm

20. Newton's rings are observed in reflected light of wavelength 5893 Å. The diameter of 15th dark ring is 0.75cm. Find the radius of curvature of the convex lens and thickness of the air film.

Solution: **Given**

Wavelength of the light (λ) = 5893 Å = 5893 $\times 10^{-8}$cm

Diameter of 15th dark ring (D_{15}) = 0.75 cm

Order of dark ring (n) = 15

Formula: The diameter of nth order dark is given by,

$$D_n^2 = 4nR\lambda$$

Determination of radius of curvature (R)

$$R = \frac{D_n^2}{4n\lambda} = \frac{(0.75)^2}{4 \times 15 \times 5893 \times 10^{-8}}$$

The radius of curvature (R) = 159 cm

Determination of thickness of air film (t)

$$t = \frac{r_{15}^2}{2R} \qquad \left(r_{15} = \frac{D_{15}}{2} = \frac{0.75}{2} = 0.375 \text{ cm}\right)$$

$$t = \frac{(0.375)^2}{2 \times 159} = 3.422$$

Thickness of air film (t) = 3.42 $\times 10^{-4}$ cm

21. Newton's rings are formed with reflected light of wavelength 5896 Å with a liquid between the plane glass plate and convex lens. The diameter of the 10th dark ring is 0.41 cm and radius of curvature of the lens is 1 meter, calculate refractive index of the liquid.

Solution: **Given**

Wavelength of light (λ) = 5896 Å = 5896 $\times$ 10^{-8} cm

Diameter of 10th dark ring (D_{10}) = 0.41 cm

Order of the dark ring (n) = 10

Radius of curvature of the lens (R) = 1 m = 100 cm

Formula: Diameter of nth order ring for liquid film is given by,

$$D_n^2 = \frac{4nR\lambda}{\mu}$$

$$\mu = \frac{4nR\lambda}{D_n^2} = \frac{4 \times 10 \times 100 \times 5893 \times 10^{-8}}{(0.41)^2}$$

Refractive index of the liquid (μ) = 1.40

22. In Newton's ring experiment if a drop of water (μ = 4/3) be placed in between the convex lens and plane glass plate, the diameter of 10^{th} dark ring is found to be 0.6 cm. Calculate the radius of curvature of the lens in contact with the glass plate. Given that wavelength of light used is λ = 6000 Å.

Solution:

Given

Refractive index of the liquid (μ) = 4/3

Diameter 10^{th} dark ring (D_{10}) = 0.6 cm

Order of dark ring (n) = 10

Wavelength of light (λ) = 6000 Å = 6000×10^{-8} cm

Formula: The diameter of n^{th} order dark ring for liquid medium is given by,

$$D_n^2 = \frac{4nR\lambda}{\mu}$$

$$R = \frac{D_n^2 \times \mu}{4n\lambda} = \frac{(0.6)^2 \times (4/3)}{4 \times 10 \times 6000 \times 10^{-8}}$$

The radius of curvature of the lens (R) = 200 cm

23. A glass microscope lens (μ = 1.5) is deposited with magnesium fluoride (μ_f = 1.38) film to increase the transmission normally incident light λ = 5800 Å. What minimum thickness should be deposited on the lens surface?

Solution:

Given

Refractive of the film (μ_f) = 1.38

Wavelength (λ) = 5800×10^{-10} m

Formula: The thickness of deposited film is given by,

$$t_{min} = \frac{\lambda}{4\mu_f} = \frac{5800 \times 10^{-10}}{4 \times 1.38}$$

$$t_{min} = 1.051 \times 10^{-7} m = 1051 \times 10^{-10} m$$

Thickness of deposited film (t_{min}) = 1051 Å

24. White light is incident at an angle of 45^0 on a soap film 4×10^{-5} cm thick. Find the wavelength of light in the visible spectrum which wavelength will be absent in the reflected light (μ= 1.2).

Solution:

Given

Angle of incidence (i) = 45^0

Thickness of soap film (t) = 4×10^{-5} cm

Refractive index of soap film (μ= 1.2).

Formula: Condition for n^{th} order darkness (reflection system)

$$2\mu t\cos r = n\lambda \quad n = 1, 2, 3 \ldots..$$

Calculation of angle of refraction (r)

We have Snell's law $$\mu = \frac{\sin i}{\sin r}$$

$$r = \sin^{-1}\left(\frac{\sin i}{\mu}\right) = \sin^{-1}\left(\frac{\sin 45}{1.2}\right) = 36.10°$$

$$\cos r = \mathbf{0.8079}$$

Calculate wavelength for various reflection order (n)

$$2\mu t \cos r = n\lambda$$

$$\lambda = \frac{2\mu t\cos r}{n}$$

For first order dark fringe, n =1

$$\lambda_1 = \frac{2\mu t\cos r}{n} = \frac{2 \times 1.2 \times 4 \times 10^{-5} \times 0.8079}{1}$$

$$\lambda_1 = 7.755 \times 10^{-5} \text{ cm}$$

$$\lambda_1 = 7755 \times 10^{-8} \text{cm}$$

For second order fringe, n = 2

$$\lambda_2 = \frac{2\mu t\cos r}{n} = \frac{2 \times 1.2 \times 4 \times 10^{-5} \times 0.8079}{2}$$

$$\lambda_1 = 3.877 \times 10^{-5} \text{ cm}$$

$$\lambda_2 = 3877 \times 10^{-8} \text{ cm}$$

For second order fringe, n = 3

$$\lambda_2 = \frac{2\mu t\cos r}{n} = \frac{2 \times 1.2 \times 4 \times 10^{-5} \times 0.8079}{3}$$

$$\lambda_3 = 2.585 \times 10^{-5} \text{ cm}$$

$$\lambda_3 = 2585 \times 10^{-8} \text{ cm}$$

$\lambda_1 = 7755 \times 10^{-8}$ cm = 7755 Å lies in visible spectrum (4000 to 7800 Å) will be absent in the reflected light.

25. A light of wavelength 5500 Å incident on thin transparent denser medium having refractive index of 1.45. Determine the thickness of thin medium, if the angle of refraction is 45^0 (Given n = 1 and reflected system).

Solution: **Given**

Order of reflection (n) = 1 and reflected system

Wavelength of light (λ) = 5500Å = 5500×10^{-10} m

Reflective index (μ) =1.45

Formula: Condition for n^{th} order minima (reflected light)

$$2\mu t\cos r = n\lambda \qquad n = 1,2,3\ldots\ldots \text{ etc.}$$

$$t = \frac{n\lambda}{2\mu t\cos r} = \frac{1 \times 5500 \times 10^{-10}}{2 \times 1.45 \times \cos 45^0} = \frac{1 \times 5500 \times 10^{-10}}{2 \times 1.45 \times 0.7071}$$

$$t = 2.68 \times 10^{-7} \text{ m}$$

Thickness of thin medium (t) = 2.68×10^{-10} m

26. Newton's rings are obtained with monochromatic light in between a flat glass plate and convex lens are viewed normally. What will be the order of the dark ring which will have double the diameter that of 40^{th} dark ring?

Solution: The diameter n^{th} order dark ring is given by,

$$D_n^2 = 4nR\lambda \tag{1}$$

The diameter of 40^{th} dark ring is given by,

$$D_{40}^2 = 4 \times 40R\lambda \tag{2}$$

Let n be the order of the dark ring whose diameter is twice that of the 40^{th} dark ring.

$$D_n = 2\,D_{40} \tag{3}$$

Equation (1) becomes, $(2D_{40})^2 = 4\,n\,R\,\lambda$

$$4D_{40}^2 = 4\,n\,R\lambda \tag{4}$$

Divide equation (2) by (4),

$$\frac{D_{40}^2}{4D_{40}^2} = \frac{4 \times 40 \times R \times \lambda}{4 \times n \times R \times \lambda}$$

Order of 40^{th} dark ring (n) = 160

27. White light falls normally on a soap film of thickness 5×10^{-5} cm and of refractive index 1.33, which wavelength in the visible region will be reflected most strongly?

Solution: **Given**

Thickness of soap film (t) = 5×10^{-5}cm

Refractive index of soap film (μ) = 1.33

Formula: We have condition for n^{th} order maxima (brightness)

$$2\mu t \cos r = (2n+1)\frac{\lambda}{2} \qquad n = 0, 1, 2, 3, \ldots\ldots\ldots\ldots\ldots$$

Calculation of wavelength (λ) for varies orders of reflection

$$\lambda = \frac{4\mu t \cos r}{(2n+1)}$$

For normal incidence, cos r = 1

For n = 0

$$\lambda_1 = \frac{4 \times 1.33 \times 5 \times 10^{-5} \times 1}{[2 \times (0) + 1]}$$

$\lambda_1 = 2.66 \times 10^{-4}$ cm

$\boldsymbol{\lambda_1 = 26600 \times 10^{-8} = 26600\ \text{Å}}$

For n = 1

$$\lambda_2 = \frac{4 \times 1.33 \times 5 \times 10^{-5} \times 1}{[2 \times (1) + 1]}$$

$\lambda_2 = 8.866 \times 10^{-5}$ cm

$\boldsymbol{\lambda_2 = 8866 \times 10^{-8} = 8866\ \text{Å}}$

For n = 2

$$\lambda_3 = \frac{4 \times 1.33 \times 5 \times 10^{-5} \times 1}{[2 \times (2) + 1]}$$

$\lambda_3 = 5.32 \times 10^{-5}$ cm

$\boldsymbol{\lambda_3 = 5320 \times 10^{-8} = 5320\ \text{Å}}$

For n = 3

$$\lambda_4 = \frac{4 \times 1.33 \times 5 \times 10^{-5} \times 1}{[2 \times (3) + 1]}$$

$\lambda_4 = 3.8 \times 10^{-5}$ cm

$\boldsymbol{\lambda_4 = 3800 \times 10^{-8} = 3800\ \text{Å}}$

λ_3 = 5320 Å lies in visible region (4000 to 7800 Å). Hence λ_3 = 5320A^0 will be the most strongly reflected wavelength.

28. A soap film of refractive index 4/3 and thickness 1.5×10^{-4} cm is illuminated by while light incident at an angle of 45^0. The light reflected by it is examined by a spectroscope in which is found a dark band corresponding to a wavelength of 5000 Å. Calculate the order of interference band.

Solution:

Given

Refractive index of soap film (μ) = 4/3 = 1.33

Thickness of soap film (t) = 1.5×10^{-4} cm

Angle of incidence (i) = 45^0

Wavelength of the light (λ) = 5000 Å = 5000×10^{-8} cm

Formula: Condition for n^{th} order dark band,

$$2\mu t \cos r = n\lambda$$

Calculation of angle of refraction (r)

We have Snell's law,

$$\mu = \frac{\sin i}{\sin r}$$

$$\sin r = \frac{\sin i}{\mu} = \frac{\sin 45^0}{1.33} = 0.5316$$

$$r = 32.11^0$$

$$\cos r = 0.8470$$

Calculation of the order of interference band (n)

$$n = \frac{2\mu t \cos r}{\lambda} = \frac{2 \times 1.33 \times 1.5 \times 10^{-4} \times 0.8470}{5000 \times 10^{-8}}$$

$$\mathbf{n = 6.75 \approx 6}$$

Order of dark band (n) = 6

29. A wedge shaped air film having angle of 40 seconds is illuminated by monochromatic light. Fringes are observed vertically through a microscope. The distance between 10 consecutive dark fringes is 1.2 cm. Find the wavelength of monochromatic light used.

Solution:

Given

Angle of wedge (α) = 40 seconds

Fringe width (β) = 1.2 cm

$$40 \text{ seconds} = \frac{40}{60 \times 60} \times \frac{\pi}{180} = 1.939 \times 10^{-4} \text{ rad}$$

Formula: $\beta = \dfrac{\lambda}{2\mu\alpha}$

For air film $\mu = 1$

$$\lambda = 2\mu\alpha\,\beta$$

$$= 2 \times 1 \times 1.2 \times \frac{40 \times \pi}{60 \times 60 \times 180}$$

$$= 3.654 \times 10^{-5}\ \text{cm} = 4654 \times 10^{-8}\ \text{cm}$$

Wavelength of monochromatic light (λ) = 4654 Å

30. Find the minimum thickness of the soap film which appear yellow (5896 Å) in reflection when it is exposed by white light at an angle of 45^0 (Given μ=1.33).

Solution:

Given

Wavelength (λ) = 5896 Å = 5896×10^{-10} m

Angle of incidence (i) = 45^0

Refractive index of soap film (μ) =1.33

Formula: Calculation of angle of refraction (r)

We have Snell's law,

$$\mu = \frac{\sin i}{\sin r}$$

$$\sin r = \frac{\sin i}{\mu} = \frac{\sin 45^0}{1.33} = 0.5316$$

$$r = 32^0\ 11'$$

$$\cos r = 0.8470$$

Condition for n^{th} order bright fringe is given by,

$$2\mu t \cos r = (2n+1)\lambda/2 \qquad n = 0, 1, 2, 3\ldots\ldots \text{ etc.}$$

$$t = \frac{\lambda}{4\mu \cos r}$$

The minimum thickness of soap film can be calculated by putting $n = 1$ and $t = t_{min}$

$$t_{min} = \frac{\lambda}{4\mu \cos r} = \frac{5896 \times 10^{-10}}{4 \times 1.33 \times 0.8470}$$

$$t_{min} = 1.308 \times 10^{-7} = 1308 \times 10^{-10}\text{m}$$

The minimum thickness of the soap film (t_{min}) = 1308 Å

31. In costume jewelry, rhinestones (made of glass with μ =1.5) are often coated with silicon monoxide (μ = 2) to make them more reflective. How thick should the coating be to achieve strong reflections for 560 nm light incident normally?

Solution: **Given**

Refractive index of coated film (μ_f) = 2

Wavelength of the light (λ) = 560 nm = 560×10^{-9} m

Formula: Condition for destructive interference is given by,

$$2\mu t \cos r = (2n+1)\lambda/2$$

The minimum thickness of coated film can be calculated by putting n = 1, t = t_{min} and for normal incidence cos r =1

$$2\mu_f t_{min} \times 1 = \frac{\lambda}{2}$$

$$t_{min} = \frac{\lambda}{4\mu_f} = \frac{560 \times 10^{-9}}{4 \times 2}$$

$$= 70 \times 10^{-9} \text{ n}$$

The thickness of coated film (t_{min}) = 70 nm

32. Newton's rings are observed in reflected light of wavelength 5 × 10^{-5} cm. the diameter of 10^{th} dark ring is 0.5 cm. Calculate radius of curvature R and thickness of film of that ring.

Solution: **Given**

Wavelength of the light (λ) = 5 $\times 10^{-5}$ cm

Diameter of 10^{th} dark ring (D_{10}) = 0.5 cm

Order of dark ring (n) =10

Formula: Diameter of n^{th} order dark ring is given by,

$$D_n^2 = 4nR\lambda$$

Calculation of radius of curvature

$$R = \frac{D_n^2}{4n\lambda} = \frac{(0.5)^2}{4 \times 10 \times 5 \times 10^{-5}}$$

Radius of curvature (*R*) = 125 cm

Calculation of thickness of thin air film (t)

The thickness of air film is given by,

$$t = \frac{r_n^2}{2R} = \frac{(0.25)^2}{2 \times 125} = 2.5 \times 10^{-4}$$

Thickness of thin air film (t) = 2.5 $\times 10^{-4}$ m

33. Newton's ring experiment the diameter of 10th dark ring changes from 1.4 cm to 1.27 cm, when a liquid is introduced between the lens and the glass plate. Calculate refractive index of the liquid.

Solution: **Given**

Diameter of 10th dark ring for air medium = D_{10} = 1.4 cm

Diameter of 10th dark ring for liquid medium (D'_{10}) = 1.27 cm

Formula: The diameter nth order dark ring is given by,

$$D_n^2 = \frac{4nR\lambda}{\mu}$$

For air film, the diameter 10th order dark ring is given by,

$$D_{10}^2 = \frac{4nR\lambda}{\mu}$$

$$D_{10}^2 = 4nR\lambda \quad (\mu_{air}=1) \tag{1}$$

For liquid film, let D'_{10} be diameter of 10th order dark ring & given by

$$D_{10}'^{\,2} = \frac{4 \times n\,R\,\lambda}{\mu} \tag{2}$$

Divide equation (1) by (2)

$$\mu = \frac{D_{10}^2}{D_{10}'^{\,2}} = \frac{(1.4)^2}{(1.27)^2} = 1.21$$

Refractive index of the liquid (μ) = 1.21

Short Answer Type Questions

1. What is meant by optics? Define the term - interference of light.
2. Define the terms- constructive and destructive interference of light.
3. Distinguish between division of wave front and division of amplitude.
4. What are the conditions to obtain sustained interference pattern of light.
5. Explain the phenomenon of interference of light. What are the necessary conditions to get clear and distinct interference fringes?
6. What do you mean by thin and thick films?
7. Explain why we see beautiful colors in thin film when it is exposed to sunlight.
8. How wedge shaped air film is formed?
9. What will be the fringe pattern if wedge shaped air film is illuminated with white light?

10. Why Newton's rings are circular?
11. Why does the fringe width decrease as order increases in Newton's rings but remain constant in a wedge shaped film?
12. Explain why in Newton's rings the rings get closer as we move away from the centre of the pattern.
13. Explain why Newton's rings are unequally spaced.
14. Explain why the system of Newton's rings observed by transmitted light is complimentary to that observed by reflected light.
15. Explain why thin film interference pattern for wedge shaped air film is parallel where as for Newton's ring it is circular.
16. What happen when a liquid is introduced between the plano-convex lens and plane glass plate in Newton's rings experiment?
17. What is meant by anti-reflection coating?
18. Why surface of lens is coated with thin transparent film?
19. What is role of refractive index of the material in the antireflection coating?
20. Explain concept of highly reflecting films.

DESCRIPTIVE ANSWER TYPE QUESTIONS

1. What is thin film? Obtain expression for the optical path difference in thin transparent film of uniform thickness due to reflected light.
2. What do you understand by the production of interference by division of amplitude method? Obtain the condition for maxima and minima of the light reflected from a thin transparent film of uniform thickness.
3. What thin film? Obtain an expression for fringe width in interference pattern of wedge shaped film.
4. Obtain expression for fringe width in wedge shaped air film.
5. Explain how to calculate thickness of thin wire using wedge shaped air film.
6. Explain formation of Newton's rings in reflected light.
7. Explain in short:- What will be the Newton's rings pattern like, if the
   (a) Monochromatic light is replaced by white light?
   (b) When plano-convex lens of larger radius of curvature (R) is used?
   (c) When a transparent liquid (μ) is placed between plano-convex lens (μ_1) and the glass plate (μ_2) such that $\mu_1 < \mu < \mu_2$.

8. How Newton's rings are obtained in the laboratory? Why do we get circular rings? Show that the radii of Newton's n^{th} dark rings are proportional to square root of natural number.

9. Prove that in Newton's rings in reflected light the radii of n^{th} order dark rings are proportional to square root of natural number.

10. Prove that in Newton's rings in reflected light the radii of n^{th} order bright rings are proportional to square root of odd natural number.

11. Explain how Newton's ring experiment is used to determine the wavelength of sodium light. Why the centre of interference pattern is dark and how can we get bright centre.

12. Explain how Newton's ring experiment is used to determine the refractive index of the liquid.

13. Discuss the testing of optical flatness of plane glass plate using interference of light.

14. What do you understand by antireflection coating? Discuss the conditions required for a thin film to act as antireflection coating.

15. Write short note on 'Testing of lens surface'.

16. Explain phenomenon of highly reflecting films.

CHAPTER **4.**

Electrodynamics

Scalar and Vector field, Physical significance of gradient, curl and divergence in Cartesian co-ordinate system, Gauss's law for electrostatics, Gauss's law for magnetostatics, Faraday's Law and Ampere's circuital law.

4.1 INTRODUCTION

The branch of physics that deals with the study of charges at rest and in motion is named as Electromagnetics which further is divided into Electrostatics (studies charges at rest – no time variation), Magneto-statics (study of charges in steady motion – no time variation), and Electrodynamics (study of charges in time varying motion that gives rise to waves that propagate and carry energy and information). Thus Electrodynamics = electromagnetic + optic.

The subject of electrodynamics is classified in to classical electrodynamics and quantum electrodynamics. The effects such as magnetism, electromagnetic radiation, and electromagnetic induction resulting due to moving charges forms what is often called classical electrodynamics.

The practical applications of these effects includes home appliances such as electric fan, table clock, microwave oven, door bell; a magnetic card reader in the door of bank's safe locker; loudspeakers; electromagnetic lifting cranes; magnetic levitation trains; electric generator, electric motor; memory storage devices in computers; telephone and mobile communication; cyclotron used to accelerate charged particles etc. It is the electrodynamics of macroscopic phenomena and was first systematically explained by the physicist James Clerk Maxwell.

Classical electrodynamics permeates into microscopic phenomenon. In 20th century physicists P. A. M. Dirac, W. Heisenberg, and W. Pauli contributed in formulation of quantum theory to explain the interaction of electromagnetic radiation with matter. This is known as quantum electrodynamics.

4.2 VECTOR ANALYSIS

In physics, some physical quantities are represented mathematically. Some concepts such as temperature, mass, distance etc. are characterized by magnitude only and algebraic sign. They are called scalars. Some concepts, for example, displacement, velocity, force, have size or magnitude, and also they have associated with them the idea of direction. Such quantities are called vectors. Both scalar and vector quantities are function of *time* and *position*.

Graphically a vector is represented by an arrow OP defining the direction and length of the arrow indicates its magnitude Fig. 4.1.

Analytically, in printed work it is represented by a letter with an arrow over it, as $\vec{A}$ or a bold face type such as **A** Fig. 4.1 and its magnitude is denoted by $|A|$ or A. Please note that the same notations for vectors and its direction will be followed for the complete chapter.

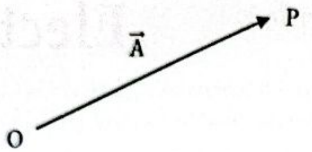


Fig. 4.1 Representation of Vector

In electromagnetic theory we often use the concept of a field. A field is a function that specifies a particular quantity to every point in a region. The field may be a vector or a scalar field that depends on the nature of the quantity under consideration.

For example, the electric potential in a region is a scalar field whereas electric or magnetic fields at any point are vector fields. The difference between a vector and a vector field is that the former is one single vector while the latter is **a distribution of vectors in space and time**. The vector field exists in all points of space and at any moment of time.

To understand various aspects of electrodynamics some mathematical operations on a scalar quantity as well as on a vector quantity are necessary. Since scalar quantities can be represented by a number alone with appropriate units, mathematical operations such as addition, subtraction, multiplication etc. can be done with simple arithmetic.

But, when doing any mathematical operations on a vector quantity one has to consider both the magnitude (size) and the direction. Thus vector analysis (a mathematical shorthand tool) is necessary to best understand and conveniently express electromagnetic concepts. Let us revise the two common operations involving vectors: the **dot product** and the **cross product**.

4.3 Dot Product

Multiplying two vectors sometimes gives a scalar quantity. This is called scalar or dot product or inner product. The dot product $\vec{A}\cdot\vec{B}$ of the vectors $\vec{A}$ and $\vec{B}$ is defined as the product of the magnitude of the vectors $\vec{A}$ and $\vec{B}$ with the cosine of the angle θ between the two vectors:

$$\vec{A}\cdot\vec{B} = AB\cos\theta \tag{4.1}$$

where $A = |A|$ and $B = |B|$ represent the magnitude of $\vec{A}$ and $\vec{B}$ respectively as in Fig. 4.2.

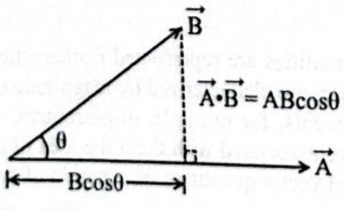


Fig. 4.2 Representation of dot product

The dot product can be positive, zero, or negative, depending on the value of cosθ. The result of $\vec{A}\cdot\vec{B}$ is always a scalar quantity.

Vector $\vec{B}$ has been split into two components, one parallel to vector $\vec{A}$ and one perpendicular to vector $\vec{A}$. Notice that the component parallel to vector $\vec{A}$ has a magnitude of $B\cos\theta$.

Therefore when we find the dot product, the result is:

(a) The magnitude of one vector, in this case A and,

(b) The magnitude of the second vector's component that runs parallel to the first vector. (That is where the cosine comes from)

If vectors $\vec{A}$ and $\vec{B}$ in three dimensions are as:

$$\vec{A} = A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}$$

and

$$\vec{B} = B_x\hat{\imath} + B_y\hat{\jmath} + B_z\hat{k}$$

where, A_x and B_x are the x-components, A_y and B_y are the y-components, and A_z and B_z are the z-components, then,

$$\vec{A}\cdot\vec{B} = A_xB_x + A_yB_y + A_zB_z \ [\because \hat{\imath}\cdot\hat{\imath} = \hat{\jmath}\cdot\hat{\jmath} = \hat{k}\cdot\hat{k} = 1;\ \hat{\imath}\cdot\hat{\jmath} = \hat{\jmath}\cdot\hat{k} = \hat{k}\cdot\hat{\imath} = 0]$$

Example: Let $\vec{A} = (\hat{\imath} + 2\hat{\jmath} - 3\hat{k})$ and $\vec{B} = B_x\hat{\imath} + B_y\hat{\jmath} + B_z\hat{k}$

Therefore $\vec{A}$ "dot" $\vec{B}$ is,

$$(\hat{\imath} + 2\hat{\jmath} - 3\hat{k})\cdot(2\hat{\imath} + 2\hat{\jmath} - \hat{k}) = 1(2) + 2(2) + (-3)(-1) = 9$$

The answer is just a number i.e. magnitude only.

Use of dot product in physics to calculate work done

Consider a situation where a constant force $\vec{F}$ (in Newtons, N) is applied to a body on a frictionless surface that causes its displacement $\vec{s}$ (in meters). The applied force increases the speed of the object. But which part of $\vec{F}$ really causes the object to increase in speed? It is $|F|\cos\theta$ as it is parallel to the displacement $\vec{s}$. Here, θ is the angle between $\vec{F}$ and $\vec{s}$.

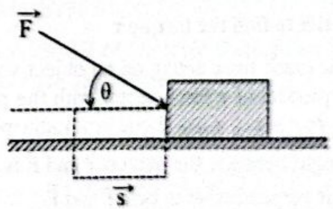


Fig. 4.3

Applying dot product, one gets ($|F|\cos\theta\,|s|$) which happens to be defined as mechanical work. Work is a type of energy and energy does not have a direction. In other words, work is a scalar or in this case a scalar product (i.e. dot product).

Thus, mathematically, the scalar product of force vector $\vec{F}$ and displacement vector $\vec{s}$ is

$$W = \vec{F}\cdot\vec{s} = |F||s|\cos\theta$$

Similarly, in electromagnetics, dot product is used to calculate:

(a) Electric potential between two points in an electric field,

(b) Magnetic flux which is the dot product of the magnetic field and the vector area.

(c) Total charge enclosed by a surface placed in an electric field.

4.4 Cross Product

Multiplying two vectors sometimes gives a vector quantity. This is called cross or vector product. This vector multiplication operation is usually used only for three dimensional vectors. Thus when we take the cross product of two vectors $\vec{A}$ and $\vec{B}$ we get back another vector $\vec{C}$ as,

$$\vec{C} = \vec{A}\times\vec{B} \tag{4.2}$$

The magnitude of vector $\vec{C}$ is,

$$C = AB\sin\theta \tag{4.3}$$

where θ is the angle separating vectors $\vec{A}$ and $\vec{B}$, as shown in Fig. 4.4.

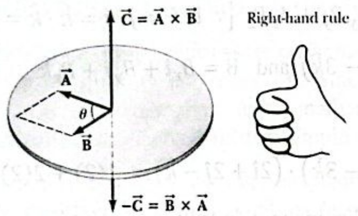


Fig. 4.4 Representation of vector cross product and right hand rule

The magnitude of $\vec{C}$, which is $AB\sin\theta$ is equal to the area of the parallelogram formed by $\vec{A}$ and $\vec{B}$. The *direction* of the vector $\vec{C}$ is perpendicular to the plane formed by vectors $\vec{A}$ and $\vec{B}$. The best way to determine this direction is to use the right-hand rule. If you curl the fingers of your right hand from vector $\vec{A}$ toward vector $\vec{B}$, then the thumb of your right hand points in the direction of $\vec{C}$.

Use of cross product in physics to find the torque τ

Torque is the measure of how much force acting on an object will cause that object to rotate about its axis. It is the cross product of a force vector with the position vector to its point of application. In the following Fig. 4.5, a force $\vec{F}$ acts at a point P and $\vec{r}$ is the vector from the point S to a point P. The angle between the vectors $\vec{r}$ and $\vec{F}$ is θ. **F** causes a torque $\boldsymbol{\tau}$ that rotates the object about an axis perpendicular to both $\vec{r}$ and $\vec{F}$.

Magnitude

The magnitude of the torque $\vec{\tau}$ about S is,

$$\tau = \vec{F}\times\vec{r} = rF_{\perp} = rF\sin\theta$$

The unit of torque is N-m.

Direction: Torque $\vec{\tau}$ is a vector in the direction of the axis of rotation and represented (by Fig. 4.5).

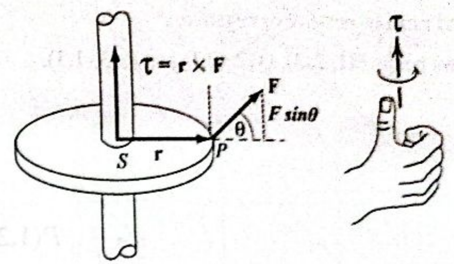


Fig. 4.5

Similarly, in electromagnetics, cross product is used to calculate:

(a) Force experienced by current carrying conductor placed in a magnetic field.

(b) The magnetic force exerted on a charge moving in a magnetic field.

4.5 Co-ordinate Systems

In electromagnetics there are many problems where we need to use different coordinate systems to describe different vector quantities in the three dimensional space. In three dimensions, the coordinate system can be specified by the intersection of three surfaces. The three coordinate surfaces may be planar or curved.

Following are the three most commonly used right handed orthogonal (three surfaces are perpendicular to each other) coordinate systems:

1. Cartesian or rectangular coordinate system
2. Cylindrical or circular coordinate system
3. Spherical or polar coordinate system

A particular coordinate system can be employed for particular problem. For example, if the geometry of the problem is cylindrical such as a wire or a cable or a cylindrical capacitor, a cylindrical coordinate system may make calculations very simple. Similarly, a problem such as electric potential of sphere can be studied using spherical coordinate system.

4.5.1 Cartesian Coordinate System

In the Cartesian coordinate system all of the three surfaces are planes. The three coordinate axes x, y, and z are mutually at right angles to each other.

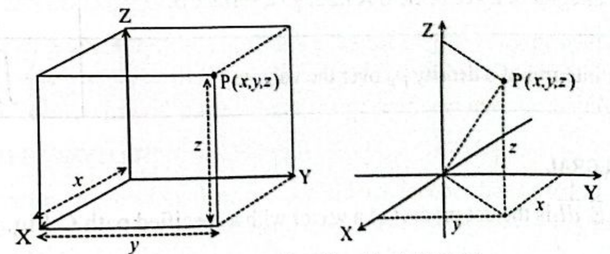


Fig. 4.6 Cartesian Coordinate System

In this coordinate system a point P in space is represented by an order triple (x, y, z). These are respectively the distances from origin to the intersection of a perpendicular dropped from the point P to the x, y, and z axes as in Fig. 4.6.

The coordinates x, y and z can be positive or negative.

Following Fig.3.7 shows points P(1, 2, 3), Q (2, -2,1) and R (3,-1,3).

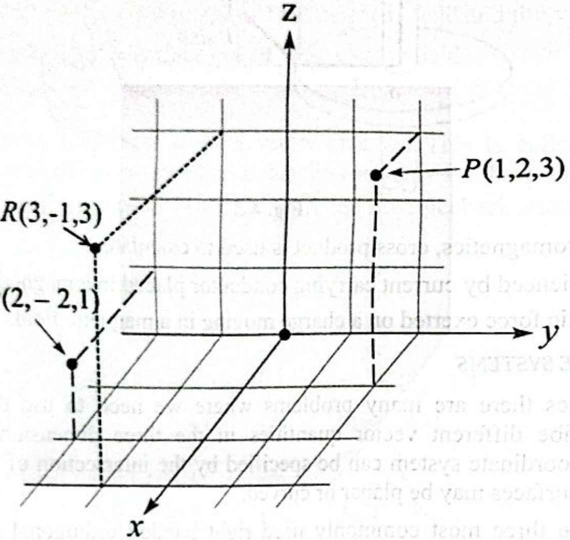


Fig. 4.7

4.6 VECTOR INTEGRATION

There are three types of integrals concerning vector quantities which are useful in deriving vector operations and in gaining an understanding of electromagnetic fields. They are listed in Table 4.1.

Table 4.1 Integrals of vector fields and volume charge density

1	Line integral of a vector field E along a prescribed path from the location a to the location b.	$\int_a^b \vec{E}.\vec{dl}$
2	Surface integral of a vector field A through a surface S.	$\iint_S \vec{A}.\vec{ds}$
3	Volume integral of a density ρ_v over the volume V.	$\iiint_V \rho_v \, .dv$

4.6.1 LINE INTEGRAL

Line integral $\int_a^b \vec{E}.\vec{dl}$ is the dot product of a vector with a specified path C Fig. 4.8.

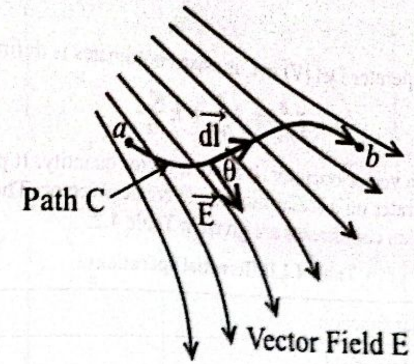


Fig. 4.8 Line Integral

It is the integral (the infinite sum) of the tangential component of a vector field $\vec{E}$ along a curve C. The concept of line integral is used to calculate the electric potential for a given electric field intensity or to calculate the work done on an object in a force field.

4.6.2 SURFACE INTEGRAL

Given a vector field $\vec{A}$ continuous in a smooth surface S, the surface integral is the flux of $\vec{A}$ through S. It is given by $\iint_S \vec{A}.\vec{ds}$ where $\vec{ds}$ is the differential surface area Fig. 4.9. The concept of surface integral is used to calculate the current density over a surface.

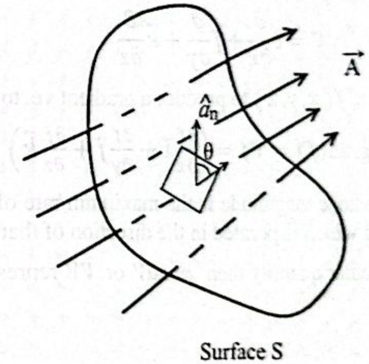


Fig. 4.9 Surface Integral

4.6.3 VOLUME INTEGRAL

Volume integral of a scalar quantity such as a volume charge density ρ_v over the volume V is given by $\iiint_V \rho_v \, .dv$. The concept of volume integral is used to calculate total charge or mass of an object in a volume provided the volume charge density or mass density is known.

4.7 VECTOR DIFFERENTIATION

In addition to vector integration, there are also differential operations used in electromagneti theory. To understand vector differentiation W. R. Hamilton introduced the vector differentia operator ∇ called Del or nabla.

4.7.1 Del Operator

The vector differential operator Del (∇) in Cartesian coordinates is defined as

$$\nabla = \vec{i}\frac{\partial}{\partial x} + \vec{j}\frac{\partial}{\partial y} + \vec{k}\frac{\partial}{\partial z} \tag{4.4}$$

Del or nabla (∇) is only a vector operator. It is not a vector quantity. It plays an important role in vector calculus. It operates on a scalar function to create a vector. The three possible vector operations of ∇ in Cartesian coordinates are given in Table 4.2.

Table 4.2 Differential operations

1	Gradient of a scalar function	∇f
2	Divergence of a vector field	$\nabla . A$
3	Curl of a vector field	$\nabla \times E$

4.7.2 Gradient

The word gradient refers to a gradual change in a quantity with respect to distance (space) i.e. rate of change of a function or derivative of a function. While rate of change (derivative) is a prominent term in two dimensional system like $y = f(x)$ in multi variable calculus or in higher dimension this same rate of change is called as gradient in vector form. A **potential gradient** is the local rate of change of potential with respect to displacement.

By definition of gradient, the Del operator,

$$\nabla = \vec{i}\frac{\partial}{\partial x} + \vec{j}\frac{\partial}{\partial y} + \vec{k}\frac{\partial}{\partial z}$$

operates on a scalar function, $f(x, y, z)$ to produce a gradient vector,

$$\text{grad}(f) = \nabla f = \left(\frac{\partial f}{\partial x}\vec{i} + \frac{\partial f}{\partial y}\vec{j} + \frac{\partial f}{\partial z}\vec{k}\right) \tag{4.5}$$

The gradient is the vector whose magnitude is the maximum rate of change of the function at the point of the gradient and which is pointed in the direction of that maximum rate of change.

If, electric potential V is a scalar quantity then $gradV$ or ∇V represents the potential gradient or electric field strength.

4.7.3 Divergence

The divergence of a vector field is a measure of how much the vector diverges or converges from that point. It is a scalar product of the vector operator ∇ and a vector field $\vec{A}$. This dot product gives a scalar. It is called divergence of $\vec{A}$ and denoted as $div\ A$ or $\nabla . A$.

If $\vec{A}$ is a vector, in Cartesian co-ordinates divergence is computed as

$$\nabla . A = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z} \tag{4.6}$$

A positive divergence (outward flow) implies the presence of a source in the given volume and negative divergence (inward flow) means a sink or drain.

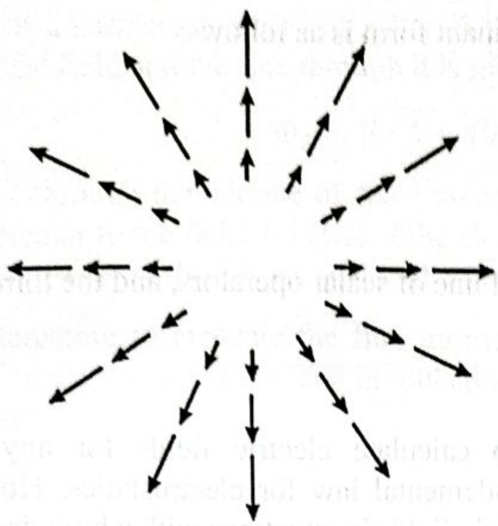


Fig. 4.10 (a) Positive Divergence

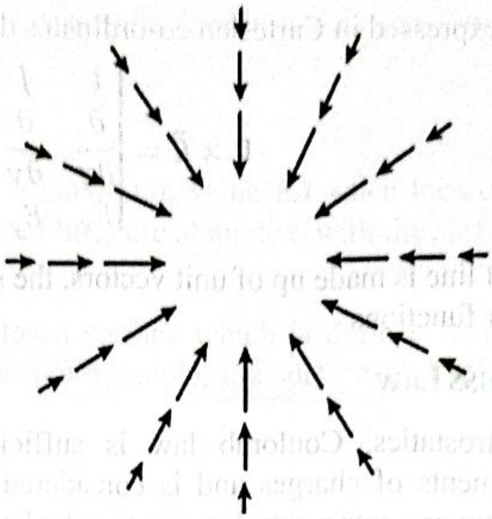


Fig. 4.10 (b) Negative Divergence

4.7.4 CURL

The cross product of the vector operator del ∇ and a vector function $\vec{F}$ gives a vector. It is called as curl of $\vec{F}$ and denoted as curl $\vec{F}$ or $curl\,\vec{F} = \nabla \times \vec{F}$. The **curl** (or rotor) is a vector operator that describes how much the vector field curls (or rotates) around the point as shown in Fig. 4.11

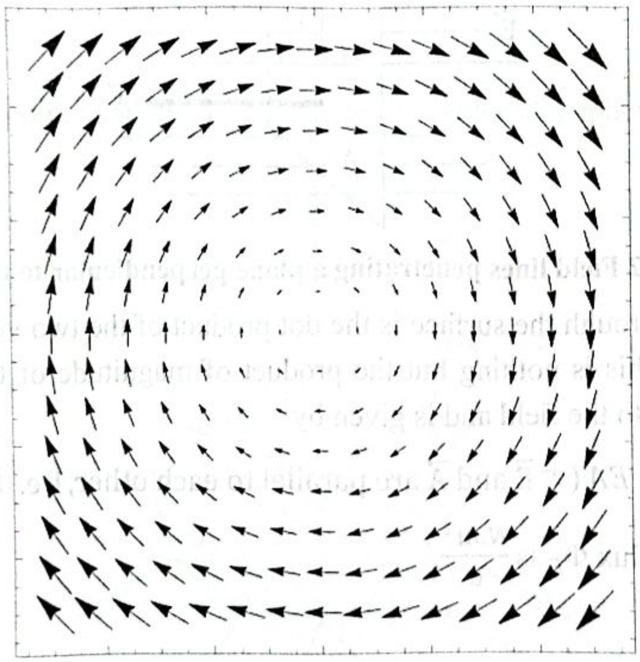


Fig. 4.11

curl of a vector field is defined as the vector field having magnitude equal to the ximum circulation at each point and orientation perpendicular to the plane of circulation. he direction of the curl is determined by the Right Hand Rule i.e. if the fingers of right hand are curled in the direction of the vector field then thumb will point in the direction of curl $\vec{F}$.

In Cartesian coordinates if i, j, k are unit vectors in the x, y, z directions then curl of electrical field $\vec{E}$ is given by,

$$\nabla \times \vec{E} = \left(\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}\right) i + \left(\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x}\right) j + \left(\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}\right) k \tag{4.7}$$

Curl $\vec{E}$ expressed in Cartesian co-ordinates determinant form is as follows:

$$\nabla \times \vec{E} = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & E_y & E_z \end{vmatrix}$$

The first line is made up of unit vectors, the second line of scalar operators, and the third line of scalar functions.

4.8 Gauss Law

In electrostatics, Coulomb law is sufficient to calculate electric fields for any static arrangements of charges and is considered a fundamental law for electrostatics. However, Gauss law presents a simpler way to calculate electric fields in situations with a high degree of symmetry. Fig. 4.12 shows an electric field that is uniform in both magnitude and direction. The lines penetrate a rectangular surface of area A which is perpendicular to the field.

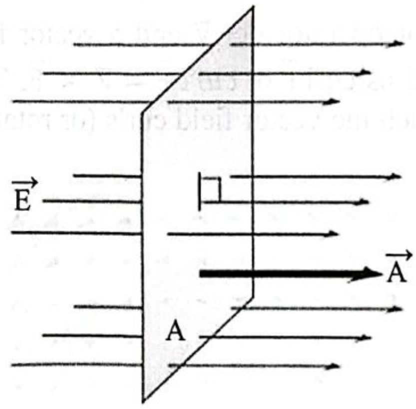


Fig. 4.12 Field lines penetrating a plane perpendicular to the field

The electric flux Φ_E through the surface is the dot product of the two vectors i.e. electric field and the area vector. This is nothing but the product of magnitude of the electric field $\vec{E}$ and surface area $\vec{A}$ parallel to the field and is given by

$$\Phi_E = \vec{E} \cdot \vec{A} = EA \; (\because \; \vec{E} \text{ and } \vec{A} \text{ are parallel to each other, i.e. } \theta = 0^o) \tag{4.8}$$

The SI unit of electric flux Φ_E is $\frac{N.m^2}{C}$.

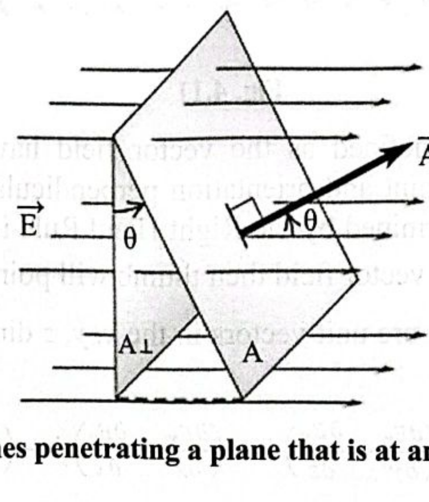


Fig. 4.13 Field lines penetrating a plane that is at an angle θ to the field

In case of a surface, as seen in the Fig. 4.13 is not perpendicular but makes some angle θ to the electric field and the flux through it is given by,

$$\Phi_E = \vec{E} \cdot \vec{A} = EA\cos\theta \tag{4.9}$$

The flux through the surface of fixed area A has a maximum value EA when the surface is perpendicular to the field. It is less if the electric field lines are at angle θ with the surface, and is zero when the surface is parallel to the field.

It is interesting to evaluate the flux through a closed surface which is defined as one that divides space into an inside and an outside region. For example, the surface of a sphere is a closed surface.

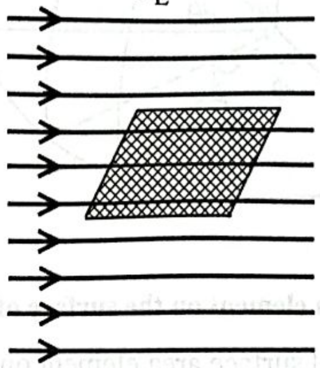


Fig. 4.14

Imagine that a positive point charge Q is situated at the center of a sphere of radius r as shown in Fig. 4.15.

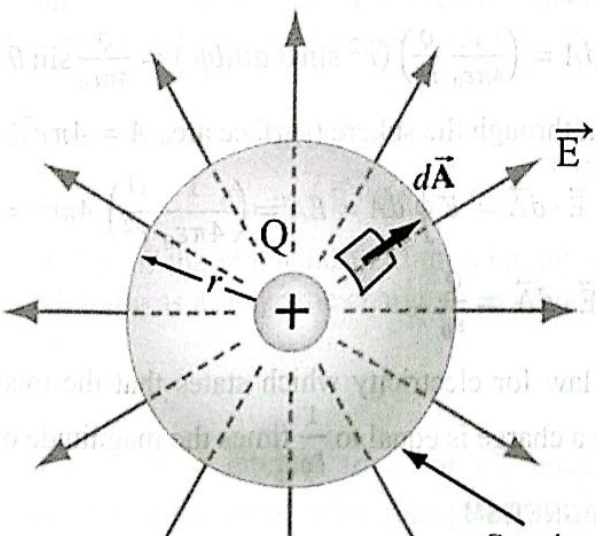


Fig. 4.15 A charge at the center of a spherical Gaussian surface

The electric field due to the charge Q is $\vec{E} = \left(\frac{Q}{4\pi\varepsilon_0 r^2}\right)\hat{r}$,

where ε_0 is the permittivity of free space and $\hat{r}$ is the unit vector.

This field points in the radial direction. The imaginary sphere of radius r that encloses the charge is called the Gaussian surface.

A small surface on the sphere is shown in Fig. 4.16.

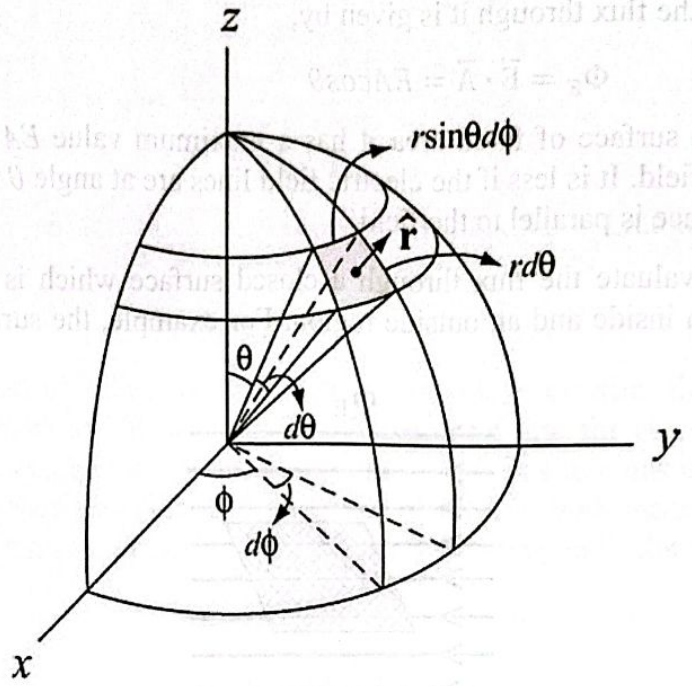


Fig. 4.16 A small area element on the surface of a sphere of radius *r*

Using spherical coordinates, a small surface area element on the sphere is given by,

$$\overrightarrow{dA} = r^2 \sin\theta \; d\theta \; d\phi \; \hat{r} \tag{4.10}$$

The net electric flux through the small area element is

$$d\Phi_E = \vec{\mathrm{E}} \cdot d\vec{\mathrm{A}} = E dA = \left(\frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\right)(r^2 \sin\theta \; d\theta d\phi\,) = \frac{Q}{4\pi\varepsilon_0}\sin\theta \; d\theta d\phi \tag{4.11}$$

And the total electric flux through the sphere (surface area $A = 4\pi r^2$) is

$$\Phi_E = \oint \vec{\mathrm{E}} \cdot d\vec{A} = E \oint dA = EA = \left(\frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\right) 4\pi r^2 = \frac{Q}{\varepsilon_0}$$

$$\therefore \; \Phi_E = \oint \vec{\mathrm{E}} \cdot d\vec{\mathrm{A}} = \frac{Q}{\varepsilon_0} \tag{4.12}$$

Eqn. 4.12 is the Gauss law for electricity which states that the total electric flux through a closed surface enclosing a charge is equal to $\frac{1}{\varepsilon_0}$ times the magnitude of the charged enclosed.

4.9 Gauss Law for Magnetism

An insulating rod having equal quantities of positive and negative charge placed on opposite ends can be cited as an example of electric dipole. On the other hand, a bar magnet with a north pole at one end and a south pole at the other end is an example of magnetic dipole. However, there is a main difference between electric and magnetic dipoles.

An electric dipole can be separated into its constituent single charges (or poles) but a magnetic dipole cannot. Each time we try to divide a magnetic dipole into separate north and south poles, we create a new pair of poles even down to the level of individual atoms. Each atom acts like a magnetic dipole having a north and a south pole and emerges to be the smallest fundamental unit of magnetic structure.

There is a Gauss Law for the electric field and for the magnetic field as well. In case of electric field if a Gaussian surface encloses a net charge Q, the electric flux Φ_E of the electric field is given by Gauss law as,

$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{Q}{\varepsilon_0}$$

Similarly, Gaussian surfaces can be constructed for the magnetic field as shown in Fig. 4.17 (a) & (b).

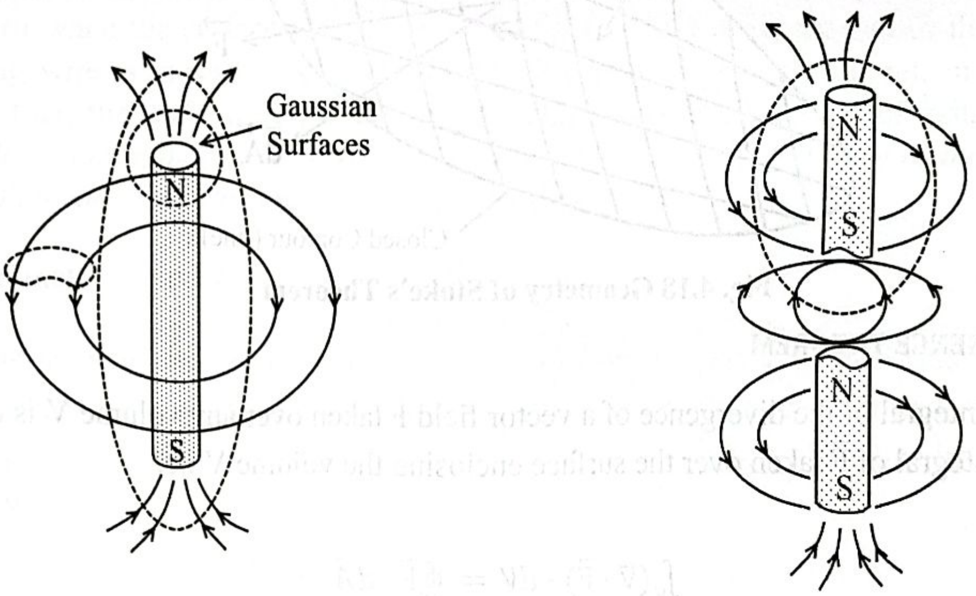


Fig. 4.17 (a) and (b) Gaussian Surface

The Gaussian surfaces, even those that cut through the bar magnet do not enclose net magnetic charge because every cut through the magnet gives a piece having both north and South Pole.

Therefore the magnetic form of Gauss law is written as,

$$\Phi_B = \oint \vec{B} \cdot d\vec{A} = 0 \tag{4.13}$$

Eqn. 4.13 states that the net flux of the magnetic field through any closed surface is zero. This is to say there is no such thing as a magnetic charge or in other words there is no such thing as a magnetic monopole.

4.10 STOKES' THEOREM

Stokes' Theorem equates a closed-line integral to a surface integral. It states that the circulation of a vector field $\vec{F}$ around a closed path C is equal to the integral of $\nabla \times \vec{F}$ over the surface S bounded by this path. It may be noted that this equality holds provided $\vec{F}$ and $\nabla \times \vec{F}$ are continuous on the surface.

$$\oint_C \vec{F} \cdot d\vec{l} = \int_S (\nabla \times \vec{F}) \cdot d\vec{A} \tag{4.14}$$

where $d\vec{l}$ is the length element of C tangent to C and $d\vec{A}$ is a directed area element normal to S.

The direction of $d\vec{A}$ is coordinated with that of $d\vec{l}$ by the right-hand rule (with the fingers curling in the direction of $d\vec{l}$, the thumb points in the direction of $d\vec{A}$). The circle on the integral signifies that the integral is computed for a closed curve C i.e. a loop.

However, the surface integral is over an open surface S whose boundary curve is C as seen in Fig. 4.18.

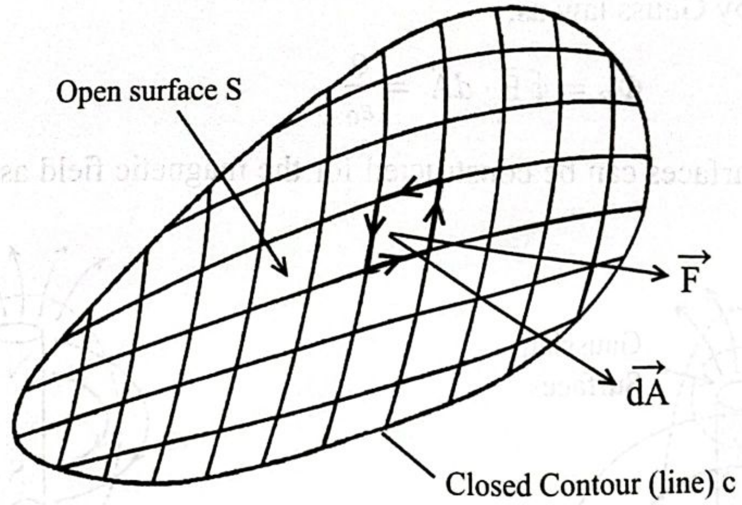


Fig. 4.18 Geometry of Stoke's Theorem

4.11 DIVERGENCE THEOREM

The volume integral of the divergence of a vector field $\vec{F}$ taken over any volume V is equal to the surface integral of $\vec{F}$ taken over the surface enclosing the volume V i.e.

$$\int_V (\nabla \cdot \vec{F}) \cdot dV = \oint_S \vec{F} \cdot d\vec{A} \tag{4.15}$$

The divergence theorem is used to convert a volume integral to a surface integral and vise versa

4.12 FARADAY'S LAW OF INDUCTION

Faraday's Law of induction is a basic law of electromagnetism predicting how a magnetic field will interact with an electric circuit to produce an electromotive force - a phenomenon called electromagnetic induction. It is the fundamental operating principle of transformers, inductors, and many types of electrical motors, generators and solenoids.

Fig. 4.19 shows edge view of a loop in a magnetic field that is uniform in both magnitude and direction.

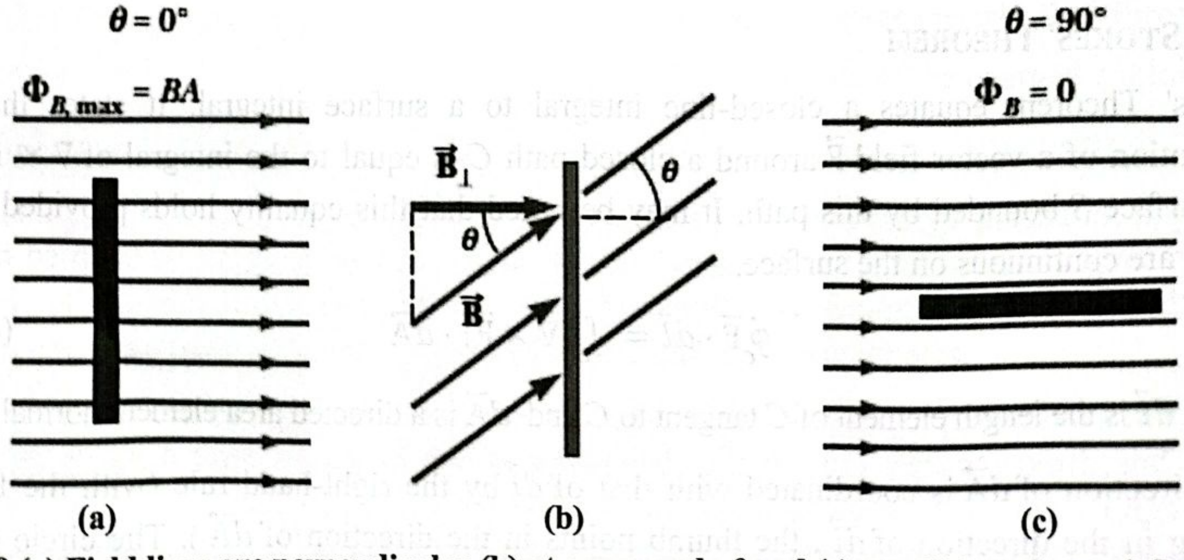


Fig. 4.19 (a) Field lines are perpendicular (b) at some angle θ and (c) parallel to plane of the loop

The magnetic flux is proportional to both the strength of the magnetic field B passing through the plane of a loop of wire and the area of the loop A. It is defined as

$$\Phi_B = BA\cos\theta \tag{4.16}$$

where θ is the angle between $\vec{B}$ and the plane of the loop.

The flux Φ_B through the surface of fixed area A has a maximum value BA when the surface is perpendicular to the field. It is less if the magnetic field lines are at angle θ with the surface and is zero when the surface is parallel to the field. If either the magnet or the loop of the conducting wire is moved an electromotive force is induced in the circuit. In terms of the magnetic flux, the *emf* induced in the circuit is given by Faraday's law of induction as: *The magnitude of the induced emf in a closed circuit is equal to the negative of the time rate of change of the magnetic flux enclosed by the circuit.*

In mathematical terms, $\mathcal{E} = -\frac{d\Phi_B}{dt}$ (4.17)

The changing magnetic field induces electric field $\vec{E}$ in the conductor. This electric field causes the charge q going around the loop. The line integral of $\vec{E}$ around a closed path represents the work done by induced electric field $\vec{E}$ per unit charge. This line integral is equal to the induced *emf* $\mathcal{E}$.

Thus, $$\oint \vec{E}.d\vec{l} = \mathcal{E} \tag{4.18}$$

Hence the Faraday's law can be restated as

$$\oint \vec{E}.d\vec{l} = -\frac{d\Phi_B}{dt} \tag{4.19}$$

4.13 Ampere's Law

Ampere's law is formulated in terms of the line integral of magnetic field $\vec{B}$ around a closed path, denoted by

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I \tag{4.20}$$

Eqn.4.20 states that for any closed loop path the line integral of magnetic field is equal to the permeability times the electric current enclosed in the loop.

Let us imagine an arbitrary closed curve called an Amperion loop as shown in Fig. 4.20. This figure shows three wires carrying current. The direction of current can be established using right hand rule which reads as: if you curl the fingers of your right hand around the integration path then your thumb points in the direction of positive current.

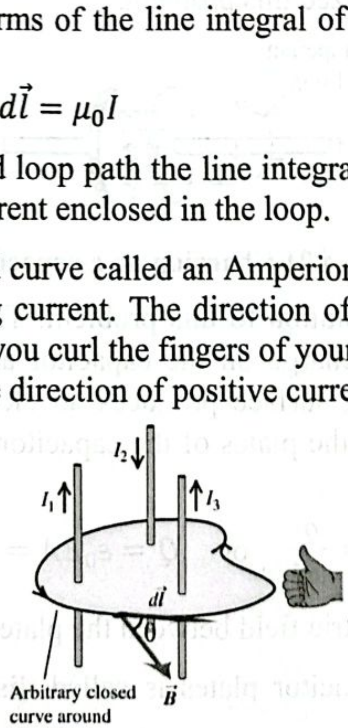


Fig. 4.20 Amperian loop

The magnetic field $\vec{B}$ at any point is the net effect of the currents in all wires. The current I in Eqn. 4.20 is the total current enclosed by the loop i.e. it is the total current carried by wires that penetrate any surface bounded by the loop. Thus the net current here is $I = I_1 - I_2 + I_3$.

The left hand side of the Eqn. 4.20 indicates that the curve be divided into small segments of length $\vec{dl}$. As we move around the loop, we evaluate the quantity $\vec{B} \cdot \vec{dl}$ and integrate all such quantities around the loop. If θ be the angle between $\vec{dl}$ and $\vec{B}$ the line integral can be written as

$$\oint \vec{B} \cdot \vec{dl} = \oint B\, dl\, cos\theta\,.$$

∴ Ampere's law for the situation shown in Fig. 3.26 can be written as

$$\oint \vec{B} \cdot \vec{dl} = \oint B\, dl\, cos\theta = \mu_0 (I_1 - I_2 + I_3) \tag{4.21}$$

4.13.1 Ampere's Law with Maxwell's Addition

Maxwell noticed a logical mistake in Ampere's law that this law could be applied only to static situations involving steady currents. Accordingly he added another source term that is a changing electric flux which extended the applicability of Ampere's law to time-dependent situations. To see the fault with Ampere's law consider a circuit with a circular parallel plate capacitor.

A current I that carry positive charge enters a left hand plate and equal amount of current I leaves the right hand plate. Therefore when capacitor becomes charged, the left plate becomes positively charged and the right plate negatively charged as shown in the Fig.4.21.

Let the Amperian loop encloses the entire left-hand capacitor plate. Therefore the left side of the Ampere's law gives the same result. However, the right side gives a very different result-namely zero- because no conducting wire pass through the surface. Thus Ampere's law appears violated. Maxwell recognized this problem.

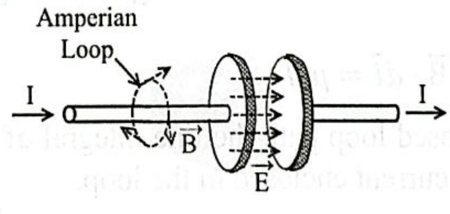


Fig. 4.21 Charging up a capacitor

Maxwell offered the following solution to this problem: The capacitor plates have a charge density $\sigma = Q/A$ where Q is the charge on the capacitor and A the area of the plates. The charge density on the conducting surface produces an electric field of strength $E = \sigma/\epsilon_0$. Thus the electric field E between the plates of the capacitor is related to the charge Q on the plates by

$$E = \frac{\sigma}{\epsilon_0} = \frac{Q}{\epsilon_0 A} \quad \text{or} \quad Q = \epsilon_0 E A = \epsilon_0 \Phi_E \tag{4.22}$$

where Φ_E is the flux of electric field between the plates.

The current flowing into the capacitor plates is called **displacement current I_D** which is related to the charge Q by

$$I_D = \frac{dQ}{dt} = \epsilon_0 \frac{d}{dt}(EA) = \epsilon_0 \frac{d\Phi_E}{dt} \tag{4.23}$$

As a result, Maxwell suggested that Ampere's law be corrected to read

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I + \mu_0 \epsilon_0 \frac{d\Phi_E}{dt} \tag{4.24}$$

Eqn.4.24 suggests that magnetic field is produced by (*a*) current through the Amperion surface and (*b*) displacement current caused due to changing electric flux. It is because this added source term $\mu_0 \epsilon_0 \frac{d\Phi_E}{dt}$, Maxwell's name is associated with the whole set of equations.

4.14 MAXWELL'S EQUATIONS

James Clerk Maxwell developed the concept of displacement current and accordingly modified the Ampere's law and knowing their importance put together the four equations i.e. Gauss law for electrostatics, Gauss law for magnetostatics, Faraday law and Ampere law in a single package. This package of four equations is known as Maxwell's equations.

Maxwell's Equations explain a diverse range of phenomena, from why a compass needle points north, to why a car starts when you turn the ignition key. They are the basis for the functioning of such electromagnetic devices as electric motors, cyclotrons, TV transmitters and receivers, telephones, fax machines, radar and microwave ovens.

An electromagnetic wave is shown in Fig. 4.22.

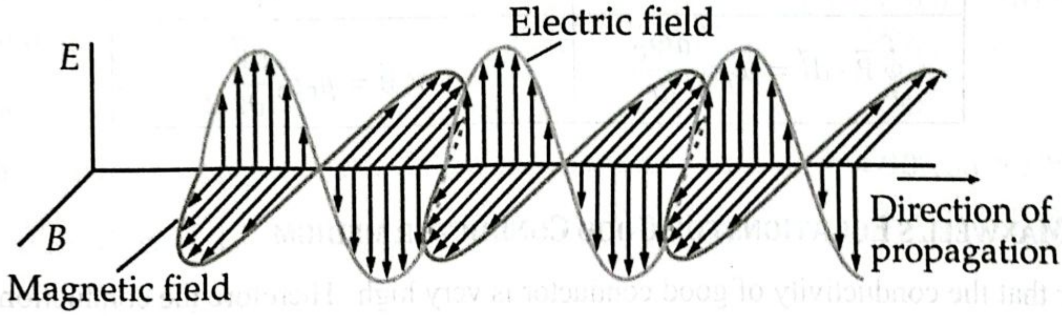


Fig. 3.22 Electromagnetic wave

Table 4.3 General Form of Maxwell's Equations

Law	Integral Form	Differential Form
Gauss Law	$\therefore \quad \Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{Q}{\varepsilon_0}$	$\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0}$
Gauss Law for Magnetism	$\Phi_B = \oint \vec{B} \cdot d\vec{A} = 0$	$\nabla \cdot \vec{B} = 0$
Faraday's law	$\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$	$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$
Ampere's law	$\oint \vec{B} \cdot d\vec{l} = \mu_0 I + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$	$\nabla \times \vec{B} = \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$

4.14.1 MAXWELL'S EQUATIONS IN EMPTY SPACE (NO CHARGE OR CURRENT)

Let us consider free space as a medium in which electric and magnetic fields are present. Since in empty space there are no charges, so the fluxes of $\vec{E}$ and $\vec{B}$ through any closed surface are equal to zero. Also, free space is a non conducting medium, so the line integrals of $\vec{E}$ and $\vec{B}$ around any closed path are related to the rate of change of flux of the other field.

Maxwell's Equations in empty space are as given below.

Table 4.4

Integral Form	Differential Form
$\oint \vec{E} \cdot d\vec{A} = 0$	$\nabla \cdot \vec{E} = \dfrac{\rho}{\varepsilon_0}$
$\oint \vec{B} \cdot d\vec{A} = 0$	$\nabla \cdot \vec{B} = 0$
$\oint \vec{E} \cdot d\vec{l} = -\dfrac{d\Phi_B}{dt}$	$\nabla \times \vec{E} = -\dfrac{\partial \vec{B}}{\partial t}$
$\oint \vec{B} \cdot d\vec{l} = \mu_0 \epsilon_0 \dfrac{d\Phi_E}{dt}$	$\nabla \times \vec{B} = \mu_0 \varepsilon_0 \dfrac{\partial \vec{E}}{\partial t}$

4.14.2 MAXWELL'S EQUATIONS FOR GOOD CONDUCTOR MEDIUM

It is clear that the conductivity of good conductor is very high. Therefore the conduction current density $\vec{J}$ is very high. Maxwell's Equations for a good conductor medium are as given below:

Table 4.5

Integral Form	Differential Form
$\oint \vec{E} \cdot d\vec{A} = 0$	$\nabla \cdot \vec{E} = 0$
$\oint \vec{B} \cdot d\vec{A} = 0$	$\nabla \cdot \vec{B} = 0$
$\oint \vec{E} \cdot d\vec{l} = -\dfrac{d\Phi_B}{dt}$	$\nabla \times \vec{E} = -\dfrac{\partial \vec{B}}{\partial t}$
$\oint \vec{B} \cdot d\vec{l} = \int_S \mathrm{J} \cdot dS$	$\nabla \times \vec{B} = \vec{J}$

Solved Problems

Example 1: Find the angle between the vectors $\vec{a} = 3\hat{\imath} + 4\hat{\jmath}$ and $\vec{b} = 6\hat{\imath} - 3\hat{\jmath} + 2\hat{k}$.

Formula: $\vec{a} \cdot \vec{b} = ab\cos\theta$

Solution: Given $\vec{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k} = 3\hat{\imath} + 4\hat{\jmath} + 0\hat{k} \Rightarrow a_1 = 3, a_2 = 4, a_3 = 0$

$$\vec{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k} = 6\hat{\imath} + 3\hat{\jmath} + 2\hat{k} \Rightarrow b_1 = 6, b_2 = -3, b_3 = 2.$$

$$\vec{a} \cdot \vec{b} = a_1b_1 + a_2b_2 + a_3b_3 = (3)(6) + (4)(-3) + (0)(2) = 18 - 12 + 0 = 6$$

$$|\vec{a}| = \sqrt{(3)^2 + (4)^2 + (0)^2} = 5$$

$$|\vec{b}| = \sqrt{(6)^2 + (-3)^2 + (2)^2} = \sqrt{49} = 7$$

$$\because \cos\theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}||\vec{b}|} = \frac{6}{(5)(7)} = \frac{6}{35}$$

$$\therefore \theta = \cos^{-1}\left(\frac{6}{35}\right) = 80°9'$$

Example 2: Find the value of $'a'$ for which the vectors $3\hat{\imath} + 2\hat{\jmath} + 9\hat{k}$ and $\hat{\imath} + a\hat{\jmath} + 3\hat{k}$ are perpendicular.

Formula: $\vec{A} \cdot \vec{B} = AB\cos\theta$

Solution: Vectors are perpendicular if their dot product is zero, i.e., $\theta = 90°$, so $\cos 90 = 0$

$$\vec{A} = 3\hat{\imath} + 2\hat{\jmath} + 9\hat{k}$$

$$\vec{B} = \hat{\imath} + a\hat{\jmath} + 3\hat{k}$$

So, $(3)(1) + (2)(a) + (9)(3) = 0$

$$\Rightarrow 2a + 30 = 0$$

$$\Rightarrow a = -15$$

Example 3: Given $\vec{A} = x^2y\,\vec{\imath} + (x - y)\vec{k}$, find (i) $\nabla \cdot \vec{A}$ & (ii) $\nabla \times \vec{A}$.

Solution: $\vec{A} = x^2y\,\vec{\imath} + (x - y)\vec{\boldsymbol{k}}$

$$\vec{A} = A_x\vec{\imath} + A_y\vec{\jmath} + A_z\vec{\boldsymbol{k}}$$

$$A_x = x^2y,\ A_y = 0,\ A_z = (x - y)$$

(i) $$\nabla \cdot \vec{A} = \left(\vec{\imath}\frac{\partial}{\partial x} + \vec{\jmath}\frac{\partial}{\partial y} + \vec{\boldsymbol{k}}\frac{\partial}{\partial z}\right) \cdot \left(A_x\vec{\imath} + A_y\vec{\jmath} + A_z\vec{\boldsymbol{k}}\right)$$

$$= \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\nabla \cdot \vec{A} = \frac{\partial(x^2y)}{\partial x} + \frac{\partial(0)}{\partial y} + \frac{\partial(x - y)}{\partial z}$$

$$\nabla \cdot \vec{A} = 2xy$$

(ii) $\nabla \times \vec{A} = \begin{vmatrix} \vec{\imath} & \vec{\jmath} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^2y & 0 & (x-y) \end{vmatrix}$

$$= \vec{\imath}\left(\frac{\partial(x-y)}{\partial y} - \frac{\partial(0)}{\partial z}\right) + \vec{\jmath}\left(\frac{\partial(x^2y)}{\partial z} - \frac{\partial(x-y)}{\partial x}\right) + \vec{k}\left(\frac{\partial(0)}{\partial x} - \frac{\partial(x^2y)}{\partial y}\right)$$

$$= \vec{\imath}(-1) + \vec{\jmath}(-1) - \vec{k}x^2$$

$$\nabla \times \vec{A} = -\vec{\imath} - \vec{\jmath} - x^2\vec{k}$$

Example 4: Constant forces $\vec{P} = 2\vec{\imath} - 5\vec{\jmath} + 6\vec{k}$ *and* $\vec{Q} = -\vec{\imath} + 2\vec{\jmath} - \vec{k}$ acts on a particle. Determine the work done when particle is displaced from A to B, the position vectors of A and B being $4\vec{\imath} - 3\vec{\jmath} - 2\vec{k}$ and $6\vec{\imath} + \vec{\jmath} - 3\vec{k}$ respectively.

Solution: Total force = $(2\vec{\imath} - 5\vec{\jmath} + 6\vec{k}) + (-\vec{\imath} + 2\vec{\jmath} - \vec{k})$

$$= \vec{\imath} - 3\vec{\jmath} + 5\vec{k}$$

Displacement $\overrightarrow{(AB)} = \vec{B} - \vec{A}$

$$= (6\vec{\imath} + \vec{\jmath} - 3\vec{k}) - (4\vec{\imath} - 3\vec{\jmath} - 2\vec{k}) = (2\vec{\imath} + 4\vec{\jmath} - \vec{k})$$

Work done = Force · Displacement

$$= (\vec{\imath} - 3\vec{\jmath} + 5\vec{k}).\ (2\vec{\imath} + 4\vec{\jmath} - \vec{k}) = 2 - 12 - 5 = -15$$

Work done = 15

Example 5: Find unit vector perpendicular to both of vectors

$$\vec{A} = 2\vec{\imath} - 3\vec{\jmath} + \vec{k},\ and\ \vec{B} = 7\vec{\imath} - 5\vec{\jmath} + \vec{k}$$

Solution: $\vec{A} \times \vec{B} = \begin{vmatrix} \vec{\imath} & \vec{\jmath} & \vec{k} \\ 2\vec{\imath} & -3\vec{\jmath} & \vec{k} \\ 7\vec{\imath} & -5\vec{\jmath} & \vec{k} \end{vmatrix}$

$$= (-3+5)\vec{\imath} - (+2-7)\vec{\jmath} + (-10+21)\vec{k}$$

$$= 2\vec{\imath} + 5\vec{\jmath} + 11\vec{k}$$

Unit vector of $\vec{A} \times \vec{B}$ = unit vector of $2\vec{\imath} + 5\vec{\jmath} + 11\vec{k}$

$$= \frac{2\vec{\imath} + 5\vec{\jmath} + 11\vec{k}}{\sqrt{(2)^2 + 5^2 + 11^2}} = \frac{2\vec{\imath} + 5\vec{\jmath} + 11\vec{k}}{\sqrt{4 + 25 + 121}}$$

Unit vector of $\vec{A} \times \vec{B}$ $= \frac{2\vec{\imath} + 5\vec{\jmath} + 11\vec{k}}{5\sqrt{6}}$.

Example 6: If $\emptyset = 3x^2y - y^3z^2$, find grad $\emptyset$ at the point (1, -2, -1).

Solution: grad $\emptyset = \nabla \emptyset$

$$= \left(\vec{i}\frac{\partial}{\partial x} + \vec{j}\frac{\partial}{\partial y} + \vec{k}\frac{\partial}{\partial z}\right)(3x^2y - y^3z^2)$$

$$= \vec{i}\frac{\partial}{\partial x}(3x^2y - y^3z^2) + \vec{j}\frac{\partial}{\partial y}(3x^2y - y^3z^2) + \vec{k}\frac{\partial}{\partial z}(3x^2y - y^3z^2)$$

$$= \vec{i}(6xy) + \vec{j}\,(3x^2 - 3y^2z^2) + \vec{k}\,(-2y^3z)$$

grad $\emptyset$ at the point (1, -2, -1),

$$= \vec{i}(6\,(1)(-2)) + \vec{j}\ [(3)(1)^2 - (3)(-2^2)(-1^2)] + \vec{k}(-2)[(-2)^3(-1)]$$
$$= -12\vec{i} - 9\vec{j} - 16\vec{k}$$

Example 7: Find normal unit vector normal to the surface $x^2 + 3y^2 + 2z^2 = 6$ at P (2, 0, 1).

Solution: $\left(\vec{i}\frac{\partial}{\partial x} + \vec{j}\frac{\partial}{\partial y} + \vec{k}\frac{\partial}{\partial z}\right)(x^2 + 3y^2 + 2z^2 - 6)$

$$= 2x\,\vec{i} + 6y\,\vec{j} + 4z\,\vec{k}$$

Normal vector at (2, 0, 1) is $4\,\vec{i} + 4\,\vec{k}$

Unit vector at (2, 0, 1) is

$$= \frac{4\vec{i} + 4\vec{k}}{\sqrt{(4)^2 + (4)^2}} = \frac{4\vec{i} + 4\vec{k}}{\sqrt{32}}$$

$$= \frac{4\vec{i} + 4\vec{k}}{4\sqrt{2}} = \frac{\vec{i} + \vec{k}}{4\sqrt{2}}.$$

Example 8: Calculate curl of vector $xyz\,\vec{i} + 3x^2y\,\vec{j} + (xz^2 - y^2z)\vec{k}$.

Solution: Curl $\vec{f} = \nabla \times \vec{f} = \left(\vec{i}\frac{\partial}{\partial x} + \vec{j}\frac{\partial}{\partial y} + \vec{k}\frac{\partial}{\partial z}\right) \times \vec{f}$

$$\nabla \times \vec{f} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ xyz & 3x^2y & (xz^2 - y^2z) \end{vmatrix}$$

$$= \vec{i}\left[\frac{\partial}{\partial y}(xz^2 - y^2z) - \frac{\partial}{\partial z}(3x^2y)\right] - \vec{j}\left[\frac{\partial}{\partial x}(xz^2 - y^2z) - \frac{\partial}{\partial z}(xyz)\right]$$
$$+ \vec{k}\left[\frac{\partial}{\partial x}(3x^2y) - \frac{\partial}{\partial y}(xyz)\right]$$

$$= \vec{i}\,(-2yz) - \vec{j}\,(z^2 - xy) + \vec{k}\,(6xy - xz)$$

$$\nabla \times \vec{f} = -2yz\,\vec{i} + (xy - z^2)\,\vec{j} + (6xy - xz)\,\vec{k}$$

SHORT ANSWER TYPE QUESTIONS

1. Explain dot product of two vectors. What is its importance in the study of electrodynamics?
2. Explain vector product of two vectors. What is its importance in the study of electrodynamics?
3. What is divergence of vector field?
4. What is curl of a vector?
5. Illustrate Gauss's law for electricity.
6. Explain Faraday's law of induction.
7. State and explain Ampere's law.
8. Find the angle between the vectors $\vec{a} = 3\hat{\imath} - 2\hat{\jmath} + 3\hat{k}$ and $\vec{b} = 5\hat{\imath} - 3\hat{\jmath} + 2\hat{k}$.
9. If $\emptyset = 2x^2y - y^3z^2$, find grad $\emptyset$ at the point (2, -2, -1).
10. Write down Maxwell's four general equations in point form.

DESCRIPTIVE ANSWER TYPE QUESTIONS

1. What is the physical significance of the electric flux through a closed surface?
2. How does Gauss's Law relate to the conservation of electric charge?
3. Why is the magnetic flux through a closed surface always zero?
4. What is the implication of the absence of magnetic monopoles?
5. What is the physical significance of the electromotive force (EMF) induced in a closed loop?
6. What is the physical significance of the magnetic field generated by a current-carrying wire?
7. Derive Gauss's Law from Coulomb's Law?
8. How does Maxwell's Correction to Ampere's Law account for the displacement current?
9. Can you think of a situation where all four Maxwell's equations are necessary to describe the electromagnetic phenomena?

CHAPTER 5.

Quantum Physics

De Broglie hypothesis of matter waves; properties of matter waves; wave packet, phase velocity and group velocity; Wave function; Physical interpretation of wave function; Heisenberg uncertainty principle; non existence of electron in nucleus; Schrodinger's time dependent wave equation; time independent wave equation; Particle trapped in one dimensional infinite potential well, Quantum Computing.

The fundamental equation of Newtonian mechanics, $F = ma = m\frac{d^2x}{dt^2}$ states that once a force is acting on a particle of mass m moving with velocity v is specified, one can always predict the state of the particle for all times. Thus Newtonian mechanics enables one to predict correctly the linear momentum, kinetic energy, potential energy, total energy etc. of the solid body whose speed is much smaller than that of light (non-relativistic) and dimensions are much greater than atomic dimensions (macroscopic).

The great triumphs for classical physics is that it has enabled us to send spacecrafts to Moon, Saturn, Mars and Jupiter but when objects of atomic or subatomic scale (microscopic domain) travel at speeds close to the speed of light (relativistic domain), Newtonian mechanics ceases to be valid.

Classical Physics also fails to explain the black body radiation, photoelectric effect, and atomic and molecular spectra. Therefore, the new theory called Quantum Mechanics (Physics) was developed through the combined efforts of the brilliant minds of the 20^{th} century. It is very interesting, weird, elegant and successful scientific theory that so far has given correct results of problems dealing with microscopic and macroscopic objects. Quantum mechanics makes use of wave picture to completely describe the behavior of a particle. It is the basis of our present understanding of all natural phenomena.

Most of the modern technologies such as Computers, Smartphones, Ultra-precise clocks, GPS, Laser, Fiber optic communication, Quantum cryptography, Quantum currency depends on the understanding of quantum physics.

5.1 Wave - Particle Duality of Radiation

Around 1800, by performing an experiment of interference of light, Thomas Young proved that light is a wave. In 1873 Maxwell with his mathematical theory predicted that light is an

electromagnetic radiation and in 1887 Hertz's wireless experiment confirmed Maxwell's result.

The theoretical explanation of black body radiation on the basis of classical physics resulted in an equation which shows that the intensity of emitted light goes on increasing continuously, meaning that an object at any temperature (even if not completely black) radiates an infinite amount of energy. This makes no sense.

In 1900 Max Planck accurately described the radiation by assuming that electro-magnetic radiation was emitted in discrete packets (or quanta) rather than a continuous wave. Each quantum has energy equal to the product of the Planck's constant and the frequency of radiation ($E = h\vartheta$). The resulting calculation not only made sense but matched what was measured experimentally.

In 1905 Einstein successfully explained photoelectric effect claiming that electro-magnetic radiation is a series of particles called photons, each with energy $h\vartheta$. These photons collide with the electrons on the metal surface causing to emit them. This shed light on Planck's relation ($E = h\vartheta$) linking energy (E) and frequency (ϑ) as arising from quantization of energy. The factor h is known as the Planck's constant. Thus when light shines through narrow slits, a look at the results suggests that it behaves as a wave but when it shines on a metal surface and examined the spray of electrons that comes off, light behaves as a particle. This behavior of light is referred to as wave-particle duality.

Though light has been shown to exhibit dual properties of wave and particle-like behavior it is difficult to accept because fundamentally a wave and a particle are different in classical physics. A wave is characterized by amplitude, wavelength, phase etc. while a particle is specified by its mass, momentum and energy.

Conceptually, a particle is considered to occupy a definite position in space at any instant of time, but a wave is necessarily extended over a relatively large region of space. However, this dual nature needs to accept for satisfactory explanation of all physical phenomena observed and quantitatively measured for electro-magnetic radiations.

5.1.1 The Photon

Einstein was awarded the 1921 Noble prize in physics for his photon theory as applied to the photoelectric effect. A photon can be described with a few of its basic properties.

1. Photon travels with the speed of light like an electromagnetic wave.
2. It has no rest mass because it can never be at rest, yet, it interacts with electrons as though it has the inertial mass $m = \frac{p}{v} = \frac{h\vartheta}{c^2}$
3. The energy it carries is related to the frequency of the electromagnetic wave by $E = h\vartheta$
4. Its linear momentum is determined by $p = \frac{h}{\lambda}$
5. The angular momentum of a micro-particle such as photon is also known as spin and is $s = 1\frac{h}{2\pi} = 1\hbar$ where $\hbar$ is the unit of angular momentum.

6. Photon can be created or destroyed when radiation is emitted or absorbed.
7. It can have particle like collision with other particles such as electron.
8. It is electrically neutral and cannot be influenced by electrical and magnetic fields.

5.2 WAVE PARTICLE DUALITY OF MATTER: DE-BROGLIE HYPOTHESIS

To provide explanation to the phenomena such as interference, diffraction and polarization, light is considered to behave as wave whereas blackbody radiation, photoelectric effect, and Compton effects could be explained on the assumption that light is composed of particles called photons. Thus light or electromagnetic radiations are imagined to exhibit wave-particle duality. It may seem logically inconsistent but it works, and is the fundamental part of quantum theory.

In 1924, Louis de-Broglie, a French theoretical physicist contributed substantially to the early development of the quantum theory by proposing that moving bodies have wave properties that complement their particle properties. His suggestion with no experimental evidence in support came partly from Bohr's theory of hydrogen atom.

The postulation that electron is supposed to follow only certain orbits around the nucleus made him think that electrons could not be considered simply as particles but periodicity must also be assigned to them.

According to his hypothesis all material particles i.e. particles of non-zero rest mass such as electrons, protons, neutrons, atoms, molecules etc. have an associated wave with them which is called a matter wave or pilot wave or de-Broglie wave.

The wavelength of matter wave is given by

$$\lambda = \frac{h}{mv} = \frac{h}{p} \tag{5.1}$$

where m is the mass, v the velocity and p the momentum of the particle.

The above equation of wavelength of matter wave can be derived using analogy of radiation as follows:

The energy of a photon associated with an electromagnetic wave of frequency ϑ is given on the basis of Planck's theory by

$$E = h\vartheta = \frac{hc}{\lambda} \tag{5.2}$$

where λ is the wavelength associated with the photon and c the velocity of electromagnetic wave in vacuum.

Also, from Einstein's energy-mass relation

$$E = mc^2 \tag{5.3}$$

From Eqns. (5.2) and (5.3) we get,

$$mc^2 = \frac{hc}{\lambda} \Rightarrow \lambda = \frac{hc}{mc^2} = \frac{h}{mc} \tag{5.4}$$

Thus the wavelength of photon is

$$\lambda = \frac{h}{mc} = \frac{h}{p} \qquad (55)$$

It means that the wavelength of a photon is specified by its momentum $p = mc$.

Now consider a material particle having mass m and moving with velocity v.

The momentum $p = mv$ of a particle associated with the wavelength λ is given by

$$\lambda = \frac{h}{mv} = \frac{h}{p} \qquad (5.6)$$

5.2.1 DE-BROGLIE WAVELENGTH OF A PARTICLE IN TERMS OF KINETIC ENERGY

The kinetic energy E of a moving particle is

$$E = \frac{1}{2}mv^2 = \frac{1}{2}\frac{m^2v^2}{m} = \frac{p^2}{2m}$$

$$\therefore \ Momentum\ p = \sqrt{2mE} \qquad (5.7)$$

Putting Eqn. (5.7) in (5.6), the de-Broglie wavelength of a material particle is

$$\lambda = \frac{h}{\sqrt{2mE}} \qquad (5.8)$$

Eqn. (5.8) gives the wavelength of a matter wave in terms of kinetic energy. Thus this equation relates the wave property of a material particle with its kinetic energy which is a particle property.

5.2.2 DE-BROGLIE WAVELENGTH OF AN ELECTRON WAVE

Consider a case of an electron accelerated through a potential difference of V volts. Its kinetic energy is

$$\frac{1}{2}mv^2 = eV$$

$$\therefore Momentum\ \ mv = \sqrt{2meV} \qquad (5.9)$$

Hence, the wavelength of the electron wave is

$$\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{h}{\sqrt{2meV}} \qquad (5.10)$$

Substituting the values of h, m, and e in Eqn. (5.10), we get

$$\lambda = \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 1.6 \times 10^{-19} \times V}}$$

$$\therefore \lambda = \frac{12.28 \times 10^{-10}}{\sqrt{V}} m = \frac{12.28}{\sqrt{V}} Å$$

5.3 de-Broglie's Model of the Atom

de-Broglie's hypothesis that a moving body behaves like a wave had no experimental evidence to corroborate. However, it received respectful attention of scientific community. He was able to show that his premise accounted for the postulate about quantization of electron angular momentum in Bohr's atomic model.

According to Bohr's assumption, only those electron orbits are allowed in which angular momentum is an integral multiple of $\frac{h}{2\pi}$, i.e.

$$mvr = \frac{nh}{2\pi} \quad \text{where } n = 1, 2, 3, ... \tag{5.11}$$

Here, $mv = p$ is the linear momentum of an electron in stationary non-radiating orbit of radius r. Thus Eqn. (5.11) can be written as

$$\therefore pr = \frac{nh}{2\pi} \tag{5.12}$$

According to de-Broglie concept, the wavelength associated with an electron as moving particle is

$$\lambda = \frac{h}{mv} = \frac{h}{p}$$

$$\therefore p = \frac{h}{\lambda} \tag{5.13}$$

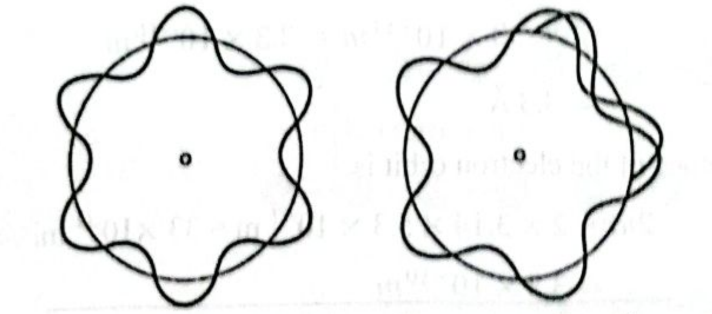
Putting Eqn. (5.13) in (5.12) we get,

$$\frac{h}{\lambda} r = \frac{nh}{2\pi}$$

$$\therefore 2\pi r = n\lambda \tag{5.14}$$

where λ is de Broglie wavelength and $n = 1, 2, 3, ...$.

Eqn. (5.14) shows that the circumference of the stationary non-radiating electron orbit is an integral multiple of de-Broglie wavelength which is also the property of standing waves. Fig. (5.1) shows the standing wave patterns of 1, 2, 3, and 4 wavelengths for Bohr's stationary orbits.



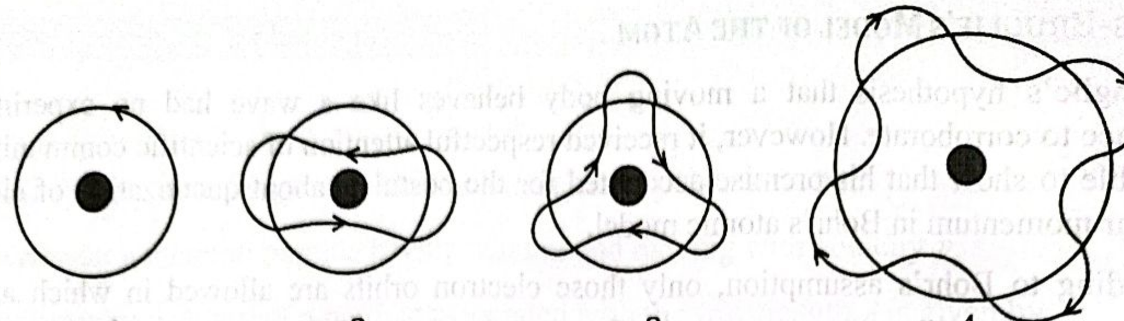


Fig: 5.1Stationary electron wave patterns

5.3.1 DE-BROGLIE WAVELENGTH OF AN ELECTRON ORBITING AROUND HYDROGEN NUCLEUS

The velocity of electron orbiting the hydrogen atom is

$$v = \frac{e}{\sqrt{4\pi\epsilon_0 mr}}$$

Now, the de-Broglie wavelength is

$$\lambda = \frac{h}{mv}$$

Putting for v in this equation we get

$$\lambda = \frac{h}{m\frac{e}{\sqrt{4\pi\epsilon_0 mr}}}$$

$$\therefore \lambda = \frac{h}{e}\sqrt{\frac{4\pi\epsilon_0 r}{m}}$$

The radius r of the electron orbit is 5.3×10^{-11} m. Putting this value of r and other constants in the above equation

$$\lambda = \frac{6.63 \times 10^{-34} J.s}{1.6 \times 10^{-19} C}\sqrt{\frac{(4\pi)(8.85 \times 10^{-12} C^2/N.m^2)(5.3 \times 10^{-11} m)}{9.1 \times 10^{-31} kg}}$$

$$\lambda = 33 \times 10^{-11} m = 3.3 \times 10^{-10} m$$

$$= 3.3 \text{ Å}$$

Now, the circumference of the electron orbit is

$$2\pi r = 2 \times 3.14 \times 5.3 \times 10^{-11} \text{ m} = 33 \times 10^{-11} \text{ m}.$$

$$= 3.3 \times 10^{-10} m$$

$$= 3.3 \text{ Å}$$

Since the wavelength of the electron orbit is exactly same as the circumference of the electron orbit, the electron orbit in a hydrogen atom is one wavelength in circumference. Thus the concept of matter waves leads to Bohr's theory of hydrogen atom.

5.4 Experimental Verification of de-Broglie Theory

The experimental confirmation of de-Broglie hypothesis of matter waves was made in 1927 by Davisson and Germer in USA and by G.P. Thomson in England. In Davisson and Germer experiment, they produced the electron beam in an electron gun in which the electrons were accelerated to the desired energy of the order of 50 eV. This beam of electrons was incident perpendicularly on the (111) plane of a Nickel crystal placed in vacuum. A schematic view of the apparatus is as shown in Fig. 5.2(a).

The Nickel crystal could be rotated about the axis of the incident beam. The incident electrons are scattered in all directions by atoms of the crystal. The detector which could be moved on an arc about the crystal receives the scattered electrons.

When the intensity of the electrons was measured as a function of scattering angle, it was found that the variation of intensity of scattered beam was not continuous with the angle of scattering. Instead, when the accelerating voltage was set at 54 V, intensity maximum occurred at the scattering angle of 50°.

Fig.5.2 (c) shows a typical polar graph of electron intensity. Had the intensity been the same at all scattering angles the curve would have been a circle centered on the point of scattering.

This striking behavior of electrons was interpreted as superposition of electron waves to give maximum intensity. The situation is like diffraction of light from a reflection grating. The spacing d between the rows of atoms resembles the spacing between the slits in the optical grating. A simplified depiction of the nickel crystal employed in Davisson and Germer experiment is shown in Fig. 5.2(a).

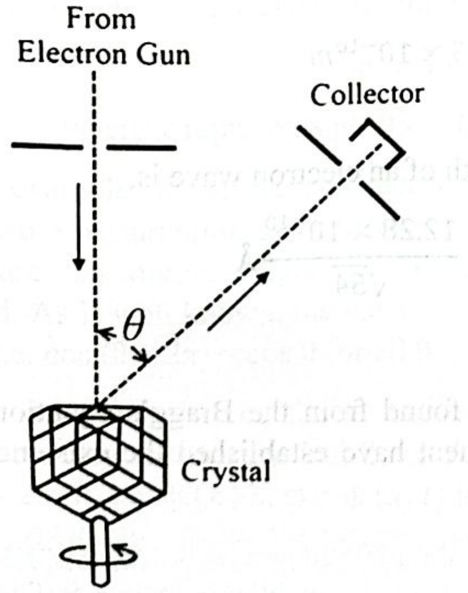


Fig. 5.2 (a) Experimental Arrangement

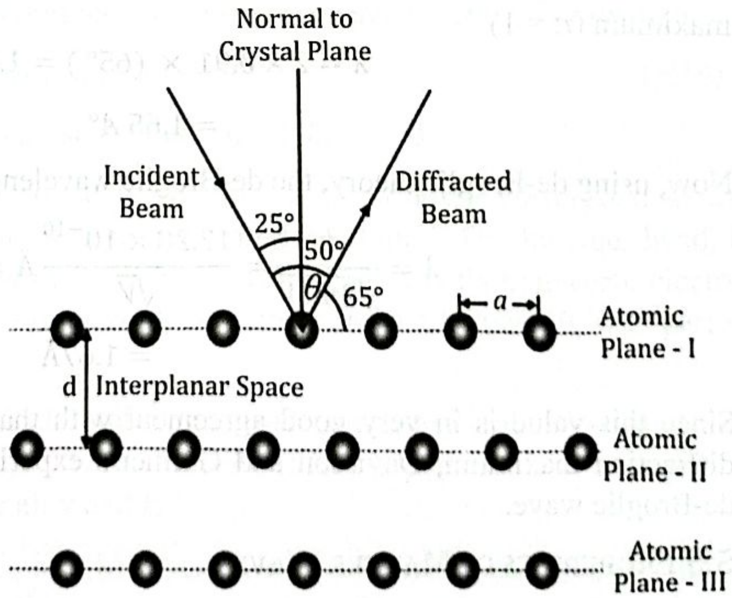


Fig. 5.2 (b) Scattering of electron beam

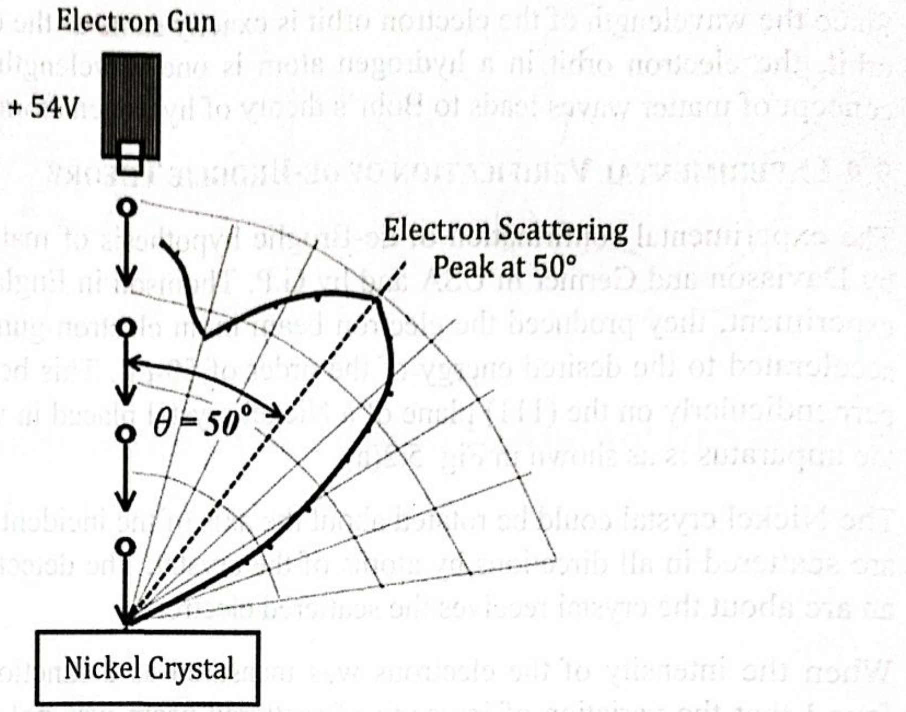


Fig. 5.2 (c) Polar Curve at 54V

Fig. 5.2 Davisson and Germer Experiment

In a typical experiment the data were as follows: The separation of atoms in the lattice $d = 0.91 \times 10^{-10}$ m, potential difference $V = 54$ and intensity maximum was observed at angle $\theta = 50°$.

Using Bragg's condition,

$$2d \sin \sin \theta \;=\; n\lambda$$

where $n = 1, 2, 3, \ldots\ldots$ is the order of diffraction, we get for first order diffraction maximum ($n = 1$)

$$\lambda = 2 \times 0.91 \times (65°) = 1.65 \times 10^{-10} m$$

$$= 1.65\, A°$$

Now, using de-Broglie theory, the de-Broglie wavelength of an electron wave is,

$$\lambda = \frac{h}{\sqrt{2meV}} = \frac{12.28 \times 10^{-10}}{\sqrt{V}} \text{Å} = \frac{12.28 \times 10^{-10}}{\sqrt{54}} \text{Å}$$

$$= 1.67\text{Å}$$

Since this value is in very good agreement with that found from the Bragg's equation for diffraction maximum, Davisson and Germer's experiment have established the existence of de-Broglie wave.

5.5 Properties of Matter Waves

(a) Larger the mass of the particle, smaller is the wavelength of the matter wave associated with it.

(b) Larger the particle velocity, smaller is the wavelength of the matter wave associated with it.

(c) The velocity of the matter waves is not constant as that of the electromagnetic radiation. It depends on the particle velocity.

(d) Matter waves are not electromagnetic waves. They are a new kind of waves.

(e) They are pilot waves in the sense that their only function is to guide the material particle in motion.

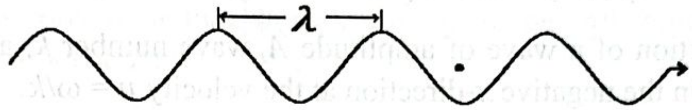


Fig: 5.3 de Broglie matter wave

(f) These waves can travel faster than light.

(g) Matter wave is not a physical phenomenon. It is rather a symbolic representation of what we know about the particle. It is a wave of probability.

5.6 WAVE FUNCTION

From the de-Broglie hypothesis of matter waves and subsequent confirmation of existence of matter wave from Davisson and Germer's experiment on electron diffraction we know that a particle or body in motion can be viewed as a wave. However, the question remains: what is waving?

A wave is defined as a disturbance in some physical system which is periodic in both space and time. And where there is wave, there is a wave function. In quantum mechanics the quantity whose variations make up matter waves is the wave function ψ (the Greek letter psi). Let us first try to understand the sound waves and light waves.

In one dimension, a wave is generally represented in terms of a wave function. For example

$$\psi(x,t) = A\cos(kx - \omega t) \tag{5.15}$$

where x represents position, t represents time, and A, k, $\omega > 0$.

For example, if we are considering a sound wave then $\psi(x, t)$ might correspond to the pressure perturbation associated with the wave at position x and time t. On the other hand, if we are considering a light wave then $\psi(x, t)$ might represent the wave's transverse electric field. As is well known, the cosine function, $\cos(\theta)$ is periodic in its argument, θ, with period 2π i.e. $\cos(\theta + 2\pi) = \cos\theta$ for all θ.

The function also oscillates between the minimum and maximum values -1 and +1, respectively as θ varies. It follows that the wave function (5.15) is periodic in x with period $\lambda = 2\pi/k$, i.e. $\psi(x+\lambda, t) = \psi(x, t)$ for all x and t.

Moreover, the wave function is periodic in t with period $T = 2\pi/\omega$ i.e. $\psi(x, t+T) = \psi(x, t)$ for all x and t. Finally the wave function oscillates between the minimum and maximum values $-A$ and $+A$ respectively, as x and t vary. ***The spatial period of the wave λ is known as its wavelength and the temporal period T is called period.***

Furthermore, the quantity A is termed the wave amplitude, the quantity k the wave number, and the quantity ω the wave angular frequency.

Note that the units of ω are radians per second. The conventional wave frequency, in cycles per second (otherwise known as hertz), is $\vartheta = 1/T = \omega/2\pi$. It follows that the maximum and by implication the whole wave propagates in the positive x-direction at velocity $v = \omega/k$. Analogous reasoning reveals that

$$\psi(x,t) = A\cos(-k\,x - \omega\,t) = A\cos(k\,x + \omega\,t) \qquad (5.16)$$

is the function of a wave of amplitude A, wave number k, and angular frequency ω, which propagates in the negative x-direction at the velocity $v = \omega/k$.

As stated above the quantity with which quantum mechanics is concerned is the wave function ψ of a particle. The wave function $\psi(x, t)$ specifies at each moment of time, the physical state of a point like particle moving in one dimension. Usually the wave functions are complex with both real and imaginary parts. Thus,

wave function $\qquad \psi = A + iB$

where A and B are real functions. The complex conjugate ψ^* of ψ is obtained by replacing i ($i = \sqrt{-1}$) by $-i$.

Complex conjugate $\qquad \psi^* = A - iB$, and

$$|\psi|^2 = \psi^*\,\psi = A^2 - i^2B^2 = A^2 + B^2 \qquad (\because i^2 = -1)$$

Hence, $\qquad |\psi|^2 = \psi^*\,\psi$ is always a positive real quantity.

The wave function $\psi(x,t)$ and its complex conjugate $\psi^*(x,t)$ provide a concrete meaning in the macroscopic physical world and therefore are the focal point of quantum mechanics. The product $\psi(x,t)\psi^*(x,t)$ is perceived as the window of quantum mechanics to the real world.

5.7 INTERPRETATION OF WAVE FUNCTION Ψ OF A MATTER WAVE

Max Born who was a pioneer of new quantum mechanics proposed the concept that the wave function $\psi(x, t)$ of a particle is related to the probability of finding the particle. For that he began with Einstein's idea. Einstein, in order to make logical the duality of particles (photons) and waves interpreted the square of the amplitude A^2 of optical wave as probability density for occurrence of photons.

In case of a matter wave the assumption is that a fictitious wave guides the motion of the material particle. The value of a wave function associated with a moving particle at a particular point x in space at time t is related to the likelihood of finding the particle there at that time.

The probability that something is present at a certain place at a given time must lie between 0 and 1. Probability 0 means the particle is definitely not there and probability 1 signifies that the particle is definitely there. An intermediate value of probability, say 0.3 means there is 30% chance of finding the particle. Though, the amplitude of a wave can be positive as well as negative, the negative probability is meaningless.

Thus the wave function $\psi(x, t)$ cannot be an observable quantity. It implies that the wave function ψ itself has no direct physical significance. The most reasonable physical interpretation of wave function is that the square of the absolute value of $|\psi(x,t)|^2 = \psi^*(x,t)\psi(x,t)$ which is always a real number, gives the probability per unit volume of finding the particle at position x at the time t.

5.7.1 NORMALIZATION

If the wave function ψ is to describe the particle properly then the integral of $|\psi(x,t)|^2$ over all space must be finite. It is convenient that $|\psi(x,t)|^2$ is equal to the probability density of finding the particle rather than proportional to it. Thus,

$$\int_{-\infty}^{\infty} \psi^*\psi \, dV = 1 \tag{5.17}$$

A wave function that obeys Eqn. (5.17) is said to be normalized.

5.8 WAVE PACKET: GROUP VELOCITY AND PHASE VELOCITY

A pure sine wave has neither a beginning nor an end i.e. it extends from $-\infty$ to $+\infty$ and is completely unlocalized as shown in Fig. (1.4a).

Fig: 5.4 (a) Sine wave

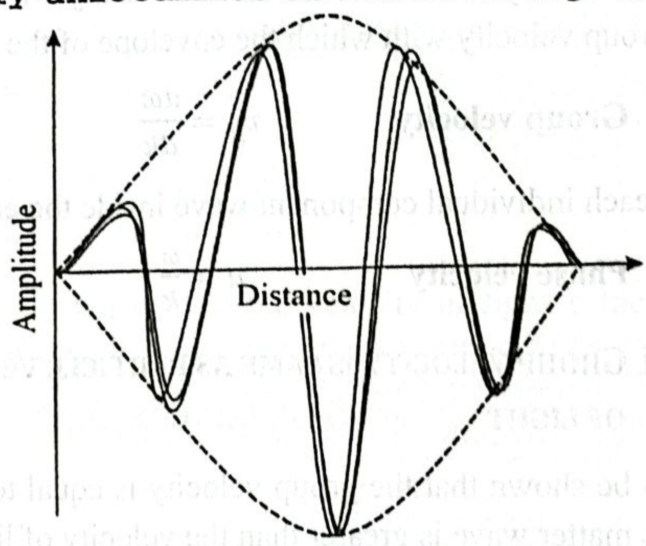


Fig: 5.4 (b) Wave Packet

An electron is though confined (localized) within the diameter of an atom its exact location remains elusive. To describe such a situation, the quantum theory which argues that a moving particle like an electron has wave properties, uses the method of a wave packet or wave group.

A **wave packet** can be looked upon as superposition of a large number of waves which interfere constructively over only a small region of space where the particle is located giving the resultant wave a large amplitude and interfere destructively far from the particle, so that the resultant wave has a small amplitude and decreases rapidly to zero as shown in Fig. (5.4b). This is in accordance with the quantum mechanical interpretation of the absolute square of the amplitude: it has something to do with probability for something to happen.

The wave packet moves with its own velocity called **Group velocity** v_g whereas the individual wave forming the packet has an average velocity known as **Wave or Phase velocity** u.

To describe the wave packet let us represent the original waves as

$$\psi_1(x,t) = A\cos(k_1 x - \omega_1 t)$$

and
$$\psi_2(x,t) = A\cos(k_2 x - \omega_2 t)$$

$$\psi(x,t) = \psi_1(x,t) + \psi_2(x,t)$$

$$\therefore \psi(x,t) = A\cos(k_1 x - \omega_1 t) + A\cos(k_2 x - \omega_2 t)$$

$$= 2A\cos\cos\left(\frac{\Delta k}{2}x - \frac{\Delta\omega}{2}t\right)\cos\cos\left(\frac{k_1+k_2}{2}x - \frac{\omega_1+\omega_2}{2}t\right) \tag{5.18}$$

where $\Delta k = k_1 - k_2$ and $\Delta\omega = \omega_1 - \omega_2$.

Thus the wave packet moves along with a speed $v = \Delta\omega/\Delta k$ while the individual wave inside it moves with a speed $(\omega_1 + \omega_2)/(k_1 + k_2)$. If $\Delta\omega$ and Δk are small then the speed with which individual waves move does not differ much from v_1 or v_2.

Instead of only two waves we can take many wave numbers and generalize the case to define the group velocity with which the envelope of the wave packet moves as

Group velocity $$v_g = \frac{d\omega}{dk} \tag{5.19}$$

And each individual component wave inside the envelope moves with its wave or

Phase velocity $$u = \frac{\omega}{k} \tag{5.20}$$

5.8.1 GROUP VELOCITY IS SAME AS PARTICLE VELOCITY WHILE WAVE VELOCITY > VELOCITY OF LIGHT

It can be shown that the group velocity is equal to the particle velocity and the wave velocity of the matter wave is greater than the velocity of light.

According to Planck's theory, $E = h\vartheta$ and from de-Broglie's theory, $p = \frac{h}{\lambda}$.

Re-writing these equations we have,

$$E = h\vartheta = \frac{h}{2\pi}(2\pi\vartheta) = \hbar\omega \quad where\ angular\ frequency\ \omega = 2\pi\vartheta$$

$$and\ \ p = \frac{h}{\lambda} = \frac{h}{2\pi}\frac{2\pi}{\lambda} = \hbar k \qquad where\ propagation\ vector\ k = \frac{2\pi}{\lambda}$$

Further, energy and momentum of the particle of mass m moving with velocity v are related by

$$E = \frac{p^2}{2m}\ \ and\ \ p = mv$$

Putting $E = \hbar\omega$ and $p = \hbar k$, we get

$$\hbar\omega = \frac{\hbar^2 k^2}{2m}$$

$$\therefore \omega = \frac{\hbar^2 k^2}{2m\,\hbar} = \frac{\hbar\, k^2}{2m}$$

$$\therefore d\omega = \frac{2\hbar\, k}{2m} dk$$

Now the group velocity

$$v_g = \frac{d\omega}{dk} = \frac{p}{m} = \frac{mv}{m} = v$$

Hence Group velocity = Particle velocity, which implies that *de Broglie wave group or wave packet has the same velocity as that of the particle.*

Let us see why the velocity of matter wave i.e. phase velocity is greater than the velocity of light. As seen above the phase velocity or wave velocity is,

$$u = \frac{\omega}{k}$$

We know that $E = h\vartheta$ and $E = mc^2$. Also, $E = \hbar\omega$ and $p = \hbar k$. Also for a material particle of mass m moving with velocity v, momentum $p = mv$.

Now therefore we have,

$$u = \frac{\omega}{k} = \frac{E\,\hbar}{\hbar\, p} = \frac{E}{p} = \frac{mc^2}{mv} = \frac{c^2}{v}$$

Since the velocity v of a material particle cannot exceed the velocity of light c the wave velocity or phase velocity of a matter wave is always greater than the velocity of light.

5.8.2 DIFFERENCE BETWEEN PHASE VELOCITY AND GROUP VELOCITY

Phase velocity	Group velocity
The velocity of individual monochromatic wave travelling through the medium with which a wave packet is constructed is known as **Wave or Phase velocity** u.	The large number of waves of slightly different frequencies traveling together in the medium, they form ***wave group*** or ***wave packet***. The velocity with which wave packet travels in the medium is **Group velocity** v_g.
The expression for Phase velocity is $u = \frac{\omega}{k}$	The expression for group velocity is $v_g = \frac{d\omega}{dk}$
Phase velocity (u) is always greater than the velocity of light.	Group velocity (v_g) is equal to velocity of the particle.

5.8.3 PHASE VELOCITY IN TERMS OF DE-BROGLIE'S WAVELENGTH

Let m be the mass of the particle travelling with velocity v and V be accelerating potential.

$\therefore$ We have $\quad E = \frac{1}{2}mv^2 = eV \qquad (5.21)$

where, e is the charge of particle.

$$E = h\vartheta \tag{5.22}$$

From equations (5.21) & (5.22) we can write

$$h\vartheta = eV \tag{5.23}$$

de-Broglie's wavelength λ in terms of accelerating potential is,

$$\lambda = \frac{h}{\sqrt{2meV}}$$

$$\lambda^2 = \frac{h^2}{2meV}$$

$$eV = \frac{h^2}{2m\lambda^2} \tag{5.24}$$

From equation (5.23) and (5.24) we have,

$$h\vartheta = \frac{h^2}{2m\lambda^2}$$

$$\vartheta = \frac{h}{2m\lambda^2}$$

We known that phase velocity, $u = \vartheta\lambda$

$$Thus \quad u = \frac{h}{2m\lambda} \tag{5.25}$$

5.9 Postulates of Quantum Mechanics

The postulates of quantum mechanics are the hypotheses and cannot be proven or deduced. If no violation with nature (experiments) is found, they are called non-provable, true statements (axioms).

Following are some of the postulates of quantum mechanics.

(a) The wave function $\psi(x, y, z, t)$ describes the temporal and spatial evolution of a quantum mechanical particle. The wave function $\psi(x, t)$ describes a particle with one degree of freedom of motion.

(b) The product $\psi^*(x,t)\,\psi(x,t)$ is the probability density function of quantum mechanical particle. $\psi^*(x,t)\,\psi(x,t)dx$ is the probability for finding the particle in the interval x and $x + dx$. Therefore

$$\int_{-\infty}^{\infty} \psi^*(x,t)\,\psi(x,t)dx = 1 \ldots\ldots\ldots\ldots (Normalization\ condition)$$

If the wave function $\psi(x,t)$ satisfies the normalization condition then $\psi(x,t)$ is termed as normalized wave function. This normalization condition suggests that the particle must be situated somewhere on the x axis.

(c) Operators are substituted for dynamical variables. The operators act on the wave function, $\psi(x,t)$ which is an operand.

5.10 HEISENBERG'S UNCERTAINTY PRINCIPLE

In classical physics we have concepts of a particle at rest or an electron has trajectory. These concepts do not exist in quantum mechanics as it deals with microscopic particles such as atoms, molecules, electrons etc. According to classical mechanics, the position x as well as the momentum p_x of the moving object can be determined with accuracy at the same time. Quantum mechanics regards a moving particle as a wave group. According to it, position x and momentum p_x of the particle cannot be determined simultaneously with accuracy.

Werner Heisenberg, a German physicist states that either the position or the momentum of the particle can be determined with great accuracy but there is a fundamental limit to the accuracy with which the position and momentum can be measured at the same time. This is called uncertainty principle. If p_x is known exactly we know nothing at all about x (i.e., if $\Delta p_x = 0, \Delta x = \infty$).

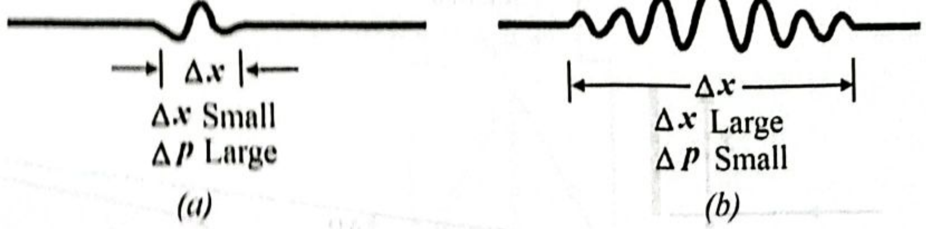
Thus, *the restriction is not on the accuracy to which x or p_x can be measured, but on the product $\Delta x.\Delta p_x$ in a simultaneous measurement of both.* In 1927 he formulated his **uncertainty principle** which states:

It is impossible, by any measurement process, to make a simultaneous determination of the position and the momentum of a particle to accuracy greater than

$$\Delta x.\Delta p_x \geq \hbar \qquad (\hbar = \frac{h}{2\pi}) \tag{5.26}$$

where $\hbar$ is called reduced Planck's constant.

To make the concept clear recall the de-Broglie equation $\lambda = \frac{h}{p}$ i.e. $p = \frac{h}{\lambda}$ in which momentum is related to the wavelength of the particle wave. Now observe the following Fig. (1.5).



(Fig: 5.5)

5.10.1 DERIVATION OF ENERGY-MOMENTUM UNCERTAINTY

The kinetic energy of micro particle of mass m is given by,

$$E = \frac{1}{2}mv^2$$

If ΔE be the uncertainty in measurement of energy then,

$$\Delta E = \frac{1}{2} m\, 2v\Delta v = v\,(m\,.\Delta v)$$

$$= \frac{\Delta x}{\Delta t}.\Delta p$$

$$\Delta E.\Delta t = \Delta x.\Delta p$$

From Uncertainty principle $\quad \Delta x.\Delta p \geq h$

$$\therefore \Delta E.\Delta t \geq h$$

The second aspect of the uncertainty principle is the limitation on the simultaneous measurement of the energy and time. Mathematically it is written as

$$\Delta E.\Delta t \geq h \tag{5.27}$$

5.10.2 Illustrations of Uncertainty Principle

(a) Electron Diffraction Experiment: Consider a long narrow slit of width Δy. Let a beam of electrons with momentum p pass through it. If Δy is comparable to the wavelength of the electron beam, then the electrons will diffract according to single slit diffraction pattern on the screen as shown in Fig. (5.6).

According to the theory of diffraction the first order diffraction minima occur if

$$\Delta y \sin\theta = \lambda \tag{5.28}$$

where θ is the angle of deviation corresponding to first minimum.

In producing the diffraction pattern on the screen all the electrons have passed through the slit but we cannot locate the exact position of the electron across the slit. Hence the uncertainty in determining the position of the electron is equal to the width Δy of the slit.

From Eqn. (5.28), we have

$$\Delta y = \frac{\lambda}{sinsin\,\theta} \tag{5.29}$$

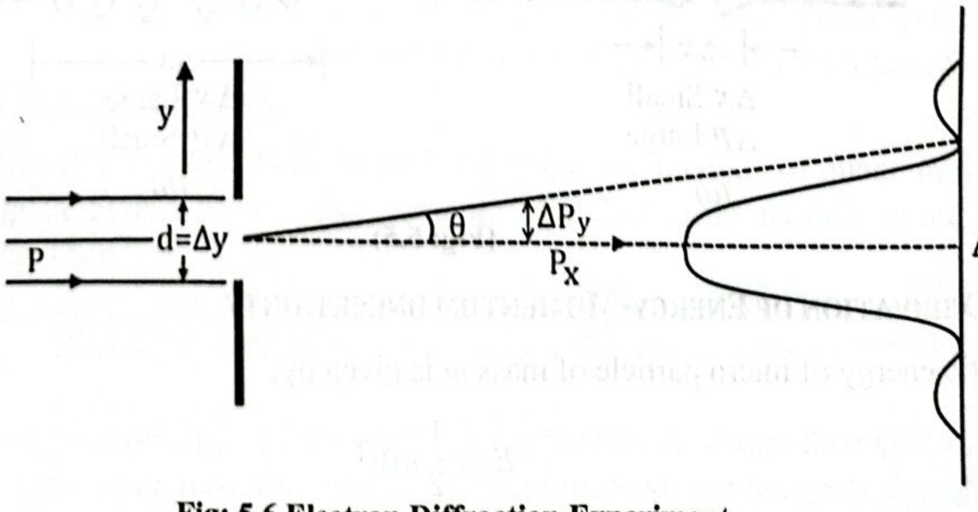


Fig: 5.6 Electron Diffraction Experiment

Before diffraction the electrons are moving along the x axis and hence they have no component of momentum along y direction. After diffraction at the slit, they are deviated from their original path to produce the diffraction pattern on the screen and have a component $psin\theta$. This y component may lie anywhere between $+psin\theta$ and $-psin\theta$. Therefore uncertainty in the y component of the momentum is

$$\Delta p_y = 2psin\theta$$

$$= 2\frac{h}{\lambda} sin\theta \tag{5.30}$$

Hence from Eqns. (5.29) and (5.30)

$$\Delta y . \Delta p_y = \frac{\lambda}{sin\, sin\, \theta} \times 2\frac{h}{\lambda} sin\theta \approx 2h$$

$$\Rightarrow \Delta y . \Delta p_y \approx h \tag{5.31}$$

A sophisticated approach shows that

$$\Delta y . \Delta p_y \geq \hbar$$

(b) Gama Ray Microscope Experiment: The resolving power of gamma ray microscope is very high. Therefore it is used to determine the position and linear momentum of an electron. The optical theory shows that the limit of resolution depends upon wavelength of light used to illuminate the object to be seen. The resolving power of the microscope is given by

$$\Delta x = \frac{\lambda}{sinsin\, \theta} \tag{5.32}$$

where Δx is the distance between the two points to be resolved, λ the wavelength of light used, and θ the semiverticale angle of cone formed by the light scattered from the illuminated object (in the present case electron) as shown in Fig. 5.7.

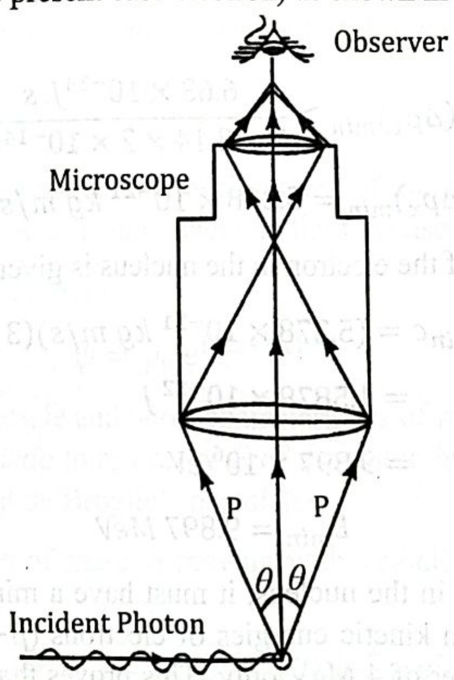


Fig: 5.7 Gama Ray Microscope Experiment

$$\Delta E = \frac{1}{2} m\, 2v\Delta v = v\,(m.\Delta v)$$

$$= \frac{\Delta x}{\Delta t}.\Delta p$$

$$\Delta E.\Delta t = \Delta x.\Delta p$$

From Uncertainty principle $\quad \Delta x.\Delta p \geq h$

$$\therefore \Delta E.\Delta t \geq h$$

The second aspect of the uncertainty principle is the limitation on the simultaneous measurement of the energy and time. Mathematically it is written as

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From Eqn. (5.28), we have

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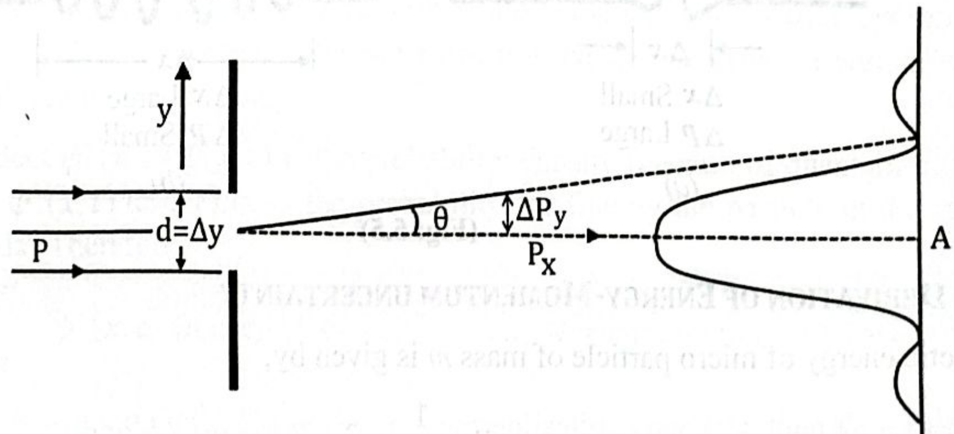

Fig: 5.6 Electron Diffraction Experiment

Since Δx is the range in which the electron can be viewed when it is disturbed by the photon it is the uncertainty in the measurement of position of the electron along x–axis which can be reduced to minimum by using shortest possible wavelength i.e. γ–rays. The photon incident upon the electron recoils from it and it enters the microscope anywhere within the angle θ and at the same time the electron is displaced. The displacement of electron contributes to the x–axis component of the momentum of the electron

$$\Delta p_x = psin\theta = \frac{h}{\lambda} sin\theta \tag{5.33}$$

$$\therefore\ \Delta x.\Delta p_x = \frac{\lambda}{sinsin\,\theta}.\frac{h}{\lambda} sin\theta \tag{5.34}$$

$$\therefore\ \Delta x.\Delta p_x = h \tag{5.35}$$

A sophisticated approach shows that

$$\Delta x.\Delta p_x \geq \hbar$$

5.10.3 Applications of Uncertainty Principle

Non-existence of Electrons in the Nucleus

The radius of the nucleus of an atom is of the order of 10^{-14} m. If electron exists inside the nucleus, the uncertainty in its position Δx is the diameter of the nucleus 2×10^{-14} m.

Heisenberg uncertainty principle is

$$(\Delta x)_{max}.(\Delta p_x)_{min} \geq \frac{h}{2\pi}$$

$$\therefore\ (\Delta p_x)_{min} \geq \frac{h}{2\pi.(\Delta x)_{max}}$$

Since $\Delta x = 10^{-14}$ meter,

$$(\Delta p_x)_{min} \geq \frac{6.63 \times 10^{-34} J.s}{2 \times 3.14 \times 2 \times 10^{-14} m}$$

$$(\Delta p_x)_{min} = 5.278 \times 10^{-21}\ kg\ m/sec$$

Now, the minimum energy of the electron in the nucleus is given by

$$E_{min} = p_{min}c = (5.278 \times 10^{-21}\ kg\ m/s)(3 \times 10^8 m/s)$$

$$= 1.5878 \times 10^{-12}\ J$$

$$= 9.897 \times 10^6\ eV$$

$$E_{min} = 9.897\ MeV$$

It means if an electron exists in the nucleus, it must have a minimum energy of about 9.897 MeV. However the maximum kinetic energies of electrons (β–particles) emitted by certain unstable nuclei are of the order of 4 MeV only. This proves that electrons do not exist in the

nucleus. Subsequently it is established that emission of $\beta-$ray is the result of transformations in the nucleus. The transformation of a neutron into a proton produces an electron.

5.11 SCHRÖDINGER'S WAVE EQUATION

In classical mechanics Newton's theory specifies the laws of motion that the macroscopic objects obeys and in quantum mechanics *Schrödinger's theory* specifies the laws of wave motion that the particles of any microscopic system obey.

Newton's laws are written down for a given situation of macroscopic particle moving under the influence of some external forces whereas the Schrodinger equation is written down for a given situation of quantum particle moving under the influence of potential energies instead of forces. Newton's laws give the trajectory of a particle but the wave function of the quantum system carries information about the wave nature of the particle that allows only to discuss the probability of finding the particle in different regions of space at a given moment in time. Schrödinger's equation explains the structure of atoms. It provides a description of physical reality in remarkable agreement with experiments.

There are two forms of the Schrödinger equation: one is Schrödinger's time dependent wave equation, which describes the wave motion of particle in the potential field that has a general dependence on space and time coordinates, and the other is time independent wave equation that describes the wave motion of particle where potential will not depend explicitly on time.

5.11.1 TIME DEPENDENT SCHRÖDINGER WAVE EQUATION IN ONE DIMENSION

Schrödinger constructed time dependent form of a wave equation for a particle of energy E moving in a potential V in one dimension. In classical mechanics the motion of a particle is described using the time-dependent position $\vec{x}(t)$ as the dynamical variable. In wave mechanics the dynamical variable is a wave function ψ.

Let us start with a wave traveling in x-direction with velocity 'u' having wave function ψ. The classical wave equation (general differential equation) of this wave is given by,

$$\frac{\partial^2\psi}{\partial x^2} = \frac{1}{u^2}\frac{\partial^2\psi}{\partial t^2} \tag{5.36}$$

where the function ψ represents the displacement of the medium in case of mechanical waves and the components of electric and magnetic field in case of electromagnetic waves.

The general solution of above equation is,

$$\psi = \psi_0\, e^{i(kx-\omega t)} \tag{5.37}$$

Schrödinger combined the particle and wave characteristics of matter by adopting the classical definition for the *non-relativistic* total energy E of a particle having momentum p, rest mass m, and potential energy V, and de Broglie's postulates.

The total energy of a particle of mass m moving with velocity v along x-axis in a potential energy field $V(x,t)$ is:

$$Total\ Energy\ E = (K.E + P.E) = \frac{1}{2}mv^2 + V(x,t)$$

$$E = \frac{p^2}{2m} + V(x,t) \tag{5.38}$$

where $V(x,t)$ is a time-dependent potential.

Schrödinger chose the energy expression of Eqn. (5.38) since it represents fundamental principle of classical physics of *conservation of energy*, which is applicable in non classical phenomena such as Einstein's relativity, Photoelectric Effect, Compton Effect, and the Bohr model of the hydrogen atom.

The wavelength of the de Broglie wave associated with this particle is given by,

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

Let us introduce the ***wave number k*** in terms of momentum of the particle,

$$k = \frac{2\pi}{\lambda} = \frac{2\pi p}{h} = \frac{p}{\hbar} \qquad \left(\because \lambda = \frac{h}{p} \;\&\; \hbar = \frac{h}{2\pi}\right)$$

and ***angular frequency ω*** of this wave in terms of particle energy,

$$E = h\vartheta = \frac{h}{2\pi} \times 2\pi\vartheta$$

$$\therefore\ E = \frac{h}{2\pi} \times \omega$$

$$\therefore\ \omega = 2\pi\frac{E}{h} = \frac{E}{\hbar}$$

Substituting $k = \frac{p}{\hbar}$ & $\omega = \frac{E}{\hbar}$ in Eqn. (5.37) we get,

$$\psi = \psi_0\, e^{i\left(\frac{px - Et}{\hbar}\right)} \tag{5.39}$$

Partial differentiating equation (5.39) with respect to '*t*',

$$\frac{\partial\psi}{\partial t} = \left(-\frac{iE}{\hbar}\right) \psi_0\, e^{i\left(\frac{px - Et}{\hbar}\right)} = \left(-\frac{iE}{\hbar}\right)\psi$$

$$E\psi = \left(-\frac{\hbar}{i}\right)\frac{\partial\psi}{\partial t} = i\hbar\frac{\partial\psi}{\partial t} \tag{5.40}$$

From double partial differentiation of equation (5.39) with respect to '*x*' we get,

$$\frac{\partial\psi}{\partial x} = \left(\frac{ip}{\hbar}\right) \psi_0\, e^{i\left(\frac{px - Et}{\hbar}\right)} = \left(\frac{ip}{\hbar}\right)\psi$$

$$\frac{\partial^2\psi}{\partial x^2} = \left(\frac{ip}{\hbar}\right)^2 \psi_0\, e^{i\left(\frac{px - Et}{\hbar}\right)} = \left(-\frac{p^2}{\hbar^2}\right)\psi$$

$$p^2\psi = -\hbar^2\frac{\partial^2\psi}{\partial x^2} \tag{5.41}$$

Multiplying both sides of equation (5.38) by wave function $\psi(x,t)$,

$$E\psi(x,t) = \frac{p^2}{2m}\psi(x,t) + V(x,t)\psi(x,t) \tag{5.42}$$

Substituting equation (5.40) & (5.41) in (5.42), we get,

$$i\hbar\frac{\partial}{\partial t}\psi(x,t) = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi(x,t) + V(x,t)\psi(x,t)$$

$$-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi(x,t) + V(x,t)\psi(x,t) = i\hbar\frac{\partial}{\partial t}\psi(x,t) \tag{5.43}$$

The first and second term at the left hand side of above equation represent ***kinetic and potential energies*** respectively of the particle and the ***right hand side*** represents its ***total energy***. Equation (5.43) is known as **one dimensional time dependent Schrödinger equation.**

5.11.2 Time Independent Schrödinger Wave Equation in One Dimension

In many cases (and in most of the cases discussed here), the potential will not depend explicitly on time. The dependence on time and position can then be separated in the Schrödinger wave equation.

Let us start with a wave traveling in *x*-direction with velocity '*u*' having wave function 'ψ'. The classical wave equation (general differential equation) of this wave is given by,

$$\frac{\partial^2\psi}{\partial x^2} = \frac{1}{u^2}\frac{\partial^2\psi}{\partial t^2} \tag{5.44}$$

where the function ψ represents the displacement of the medium in case of mechanical waves and the components of electric and magnetic field in case of electromagnetic waves.

The general solution of above equation is,

$$\psi = \psi_0\, e^{i(kx-\omega t)} \tag{5.45}$$

where, Ψ_0 is wave constant.

Now, consider a particle of mass *m* moving with velocity *v* along *x*-axis in a potential energy field $V(x)$. The total energy of this particle is the sum of its kinetic and potential energies

$$Total\ Energy\ E = KE + PE = \frac{1}{2}mv^2 + V(x)$$

$$E = \frac{p^2}{2m} + V(x) \tag{5.46}$$

where $V(x)$ denotes that the potential energy of the particle is independent of time.

The wavelength of the de-Broglie wave associated with this particle is given by,

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

Let us introduce the ***wave number k*** in terms of momentum of the particle,

$$k = \frac{2\pi}{\lambda} = \frac{2\pi p}{h} = \frac{p}{\hbar} \quad \left(\because \lambda = \frac{h}{p} \;\&\; \hbar = \frac{h}{2\pi}\right)$$

and ***angular frequency*** ω of this wave in terms of particle energy,

$$E = h\vartheta = \frac{h}{2\pi} \times 2\pi\vartheta$$

$$\therefore\; E = \frac{h}{2\pi} \times \omega$$

$$\therefore\; \omega = 2\pi\frac{E}{h} = \frac{E}{\hbar}$$

Substituting $k = \frac{p}{\hbar}$ & $\omega = \frac{E}{\hbar}$ in Eqn. (5.45) we get,

$$\psi = \psi_0\, e^{i\left(\frac{px-Et}{\hbar}\right)} \tag{5.47}$$

Taking first order partial derivative of Eqn. (5.47) with respect to space and time,

$$\frac{\partial\psi}{\partial x} = \frac{i}{\hbar}p\psi$$

$$\textit{and} \quad \frac{\partial\psi}{\partial t} = -\frac{i}{\hbar}E\psi$$

From this derivative, we get

$$E\psi = i\hbar\frac{\partial\psi}{\partial t} \quad (\because i^2 = -1) \tag{5.48}$$

The total energy formula in Eqn. (5.46) contains the square of the momentum, so let us calculate the second order partial with respect to space:

$$\frac{\partial^2\psi}{\partial x^2} = -\frac{p^2}{\hbar^2}\psi$$

From this derivative, we get

$$p^2\psi = -\hbar^2\frac{\partial^2\psi}{\partial x^2} \tag{5.49}$$

Now multiplying Eqn.(5.46) with $\psi(x)$,

$$E\psi(x) = \frac{p^2}{2m}\psi(x) + V(x)\psi(x) \tag{5.50}$$

Substituting Eqns. (5.48) and (5.49) in Eqn. (5.50) we get,

$$E\psi(x) = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi(x) + V(x)\psi(x)$$

$$\therefore\; \frac{\partial^2}{\partial x^2}\psi(x) + \frac{8\pi^2 m}{h^2}[E - V(x)]\psi(x) = 0 \tag{5.51}$$

Eqn. (5.51) is known as ***time independent Schrödinger Equation***. It gives spatial variation of matter waves in one dimension. This equation is more restrictive than the original time-

dependent Schrödinger equation, because it assumes that the particle/ wave has a definite energy (that is, a definite ω).

5.12 Applications of Schrodinger's Equation

5.12.1 Motion of a Free Particle

Let us consider an electron moving freely in space in positive $x-$direction and not acted upon by any force. As no force is acting on the electron its potential energy V is zero.

The Schrodinger wave equation is

$$\frac{\partial^2\psi}{\partial x^2}+\frac{8\pi^2 m}{h^2}E\psi = 0 \quad (5.52)$$

Writing $\frac{8\pi^2 m}{h^2}E = k^2$ in Eqn.(5.52), we get

$$\frac{\partial^2\psi}{\partial x^2}+k^2\psi = 0 \quad (5.53)$$

A particle is a free moving particle and as such there are no boundary conditions. Therefore there are no restrictions on k. Hence energy is not quantized but all values of energy are allowed. These values can be found from

$$E = \frac{h^2}{8\pi^2 m}k^2 \quad (5.54)$$

Thus a freely moving particle possesses a continuous energy spectrum. Eqn.(5.54) represents the kinetic energy of the particle.

5.12.2 Particle Trapped in One Dimensional Infinite Well

An infinite potential well model is used to describe a particle (electron) that is free to move in a small space surrounded by impenetrable barriers. This model is a hypothetical example that demonstrates the differences between classical and quantum systems. The word one dimension indicates that the particle is moving in one dimension.

The model is also known as the infinite square well or the particle in a box. The potential well here is a very (infinitely) tall rectangular well but it is called square well because of the right-angled corners. The name particle in a box is used to emphasize that the particle can freely move around inside a given region, but has zero probability of leaving the region, just like a box. Hence, outside the box, $\psi(x) = 0$.

Consider a particle of mass m moving freely along $x-$direction in a one dimensional well as shown in Fig. (5.8). The particle is completely trapped within the well of infinite height and width a. The particle bounces back and forth between the hard walls of the well at $x = 0$ and $x = a$. The particle does not lose energy on collision with such walls, so that its total energy remains constant. From the formal point of view the potential energy V of the particle is infinite on both sides of the box whereas it has any constant value say zero for convenience within the well. Thus

$$V(x) = 0 \qquad 0 \le x \le a$$
$$V(x) = \infty \qquad x < 0, x > a \quad (5.55)$$

Though in Newtonian view the particle is traveling along a straight line bouncing between two rigid walls, in quantum view, the particle is no more pictured as a particle bouncing between

the walls but a wave (de-Broglie wave) that is trapped inside the infinite quantum well, in which there forms standing waves.

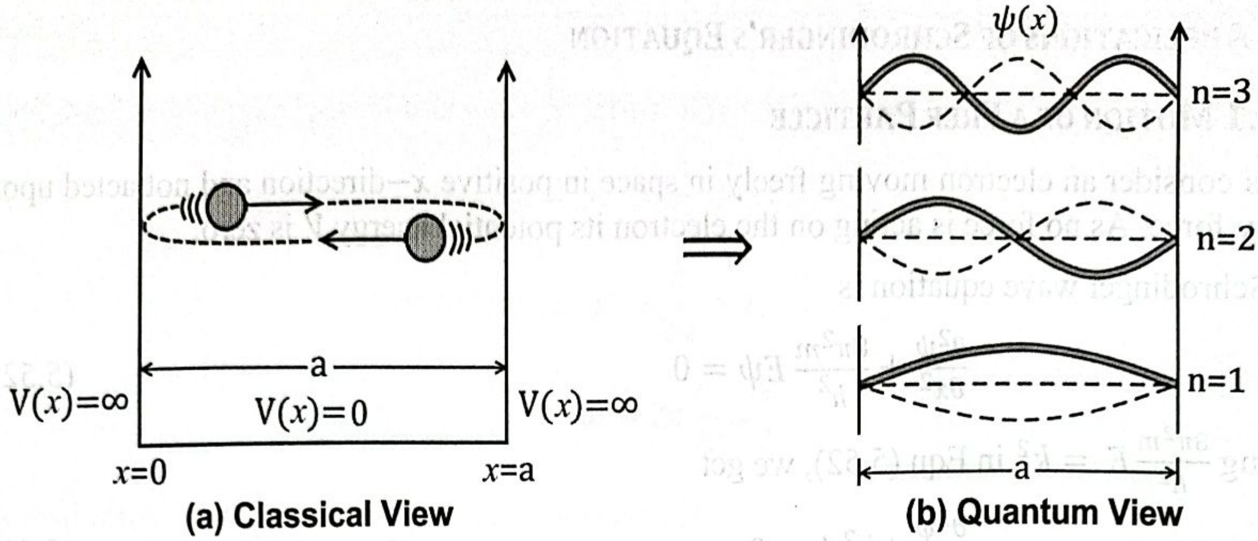

Fig. 5.8: Particle trapped in one dimensional potential well

If the walls of the box are perfectly rigid the particle must always be in the box, and the probability for finding it elsewhere must be zero. To make the probability zero everywhere outside the box, we must make $\psi = 0$ outside the box. Thus

$$\psi(x) = 0 \qquad x < 0,\ x > a \tag{5.56}$$

Within the box the Schrodinger's wave equation becomes

$$\frac{\partial^2\psi}{\partial x^2} + \frac{8\pi^2 m}{h^2} E\psi = 0 \tag{5.57}$$

Let us write

$$\frac{8\pi^2 m}{h^2} E = k^2 \tag{5.58}$$

Substituting Eqn. (5.58) in (5.57) we get

$$\frac{\partial^2\psi}{\partial x^2} + k^2\psi = 0 \tag{5.59}$$

The solution of differential equation (5.59) is,

$$\psi(x) = A\, sin\, sin\, kx + B\, cos\, cos\, kx \tag{5.60}$$

where A and B are constants to be evaluated.

The above solution is subjected to the boundary conditions that $\psi = 0$ for $x = 0$ and for $x = a$. Let us apply these boundary conditions and evaluate constants A and B.

(a) At $x = 0, \psi(0) = 0$

i.e. $0 = A\, sin\, sin\, 0 + B\, cos\, cos\, 0$

$0 = 0 + B$ $\qquad$ ($\because \cos 0^\circ = 1$)

$\therefore\ B = 0$

It means that Eqn. (5.60) becomes

$$\psi(x) = A \sin k x \tag{5.61}$$

(b) At $x = a, \psi(a) = 0$

i.e. $0 = A \sin \sin ka \Rightarrow A \sin \sin ka = 0$ (5.62)

Either $A = 0$, in which case $\psi = 0$ everywhere, $\psi^2 = 0$ everywhere, and there is no particle i.e. a meaningless solution or else $\sin ka = 0$, which is true only when $ka = \pi, 2\pi, 3\pi, \ldots$

$$\therefore ka = n\pi \quad (n = 1, 2, 3, \ldots) \tag{5.63}$$

$$\therefore k = \frac{n\pi}{a} \tag{5.64}$$

Substituting this value of k in Eqn. (5.58), we get

$$\frac{8\pi^2 m}{h^2} E = \frac{n^2\pi^2}{a^2} \tag{5.65}$$

$$\Rightarrow E_n = \frac{h^2}{8\pi^2 m} \frac{n^2\pi^2}{a^2} \tag{5.66}$$

$$\therefore E_n = \frac{n^2 h^2}{8ma^2} \quad (n = 1, 2, 3, \ldots) \tag{5.67}$$

Eqn. (5.67) gives certain discrete values called the eigen values of energy of particle for different values of n. The integer n corresponding to energy E_n is called quantum number. The values of energy E_n for which Schrodinger's steady-state equation can be solved are called eigen values and the corresponding wave functions ψ_n are called eigen functions. Thus, a particle trapped in one-dimensional infinite potential well possesses discrete values of energy.

It is seen that $E = 0$ is excluded, because in case the particle possesses zero energy inside the box, the wave function inside the box turns out to be zero which means particle no more exists inside the box, which is incorrect. Therefore, the particle can have all values of energy given by Eqn. (5.67) excluding zero value. Furthermore, the quantum number n indicates the number of nodes in the wave.

A few of the energy levels are

$E_1 = \dfrac{h^2}{8ma^2}$ lowest energy level

$E_2 = \dfrac{4h^2}{8ma^2}$ 4 times the minimum energy

$E_3 = \dfrac{9h^2}{8ma^2}$ 9 times the minimum energy

These energy levels are shown in right hand-side of Fig. 5.8.

5.13 QUANTUM COMPUTING

The field of quantum computing emerged in the 1980s. It was discovered that certain computational problems could be tackled more efficiently with quantum algorithms than with their classical counterparts

5.13.1 QUANTUM MECHANICAL TUNNELLING

Quantum mechanical tunnelling is the phenomenon in which particles pass through a barrier that they should not be able to cross. Consider rolling a ball up a hill; typically, the ball requires sufficient energy to overcome the hill. In quantum mechanics, even if the particle lacks sufficient energy, it has a chance of "tunnelling" through the hill and appearing on the other side. This occurs because particles such as electrons have wave-like properties that allow them to penetrate barriers at the quantum level, which classical physics cannot explain.

WHY QUAMTUM COMPUTING?

Quantum computing addresses fundamental limitations of classical computing. Classical computing encodes information in binary bits (0 or 1). Transistors in classical computers have shrunk to such a small size that they generate significant heat, posing a risk of failure for silicon-based technology. Additionally, at such small scales, electrons can tunnel through micro-thin barriers between wires, disrupting computing signals. Quantum computing, however, uses quantum bits (qubits) that can represent both 0 and 1 simultaneously due to superposition. This, combined with entanglement, allows quantum computers to process vast amounts of information simultaneously and solve certain problems much faster than classical computers. Moreover, quantum computing operates on different principles that can potentially reduce the heat issues associated with further miniaturization in classical computing.

PRINCIPLE

Quantum computing uses qubits that can be in multiple states at once (superposition) and can be linked together (entanglement). Quantum interference uses the way quantum states combine to increase the chances of correct results and decrease errors. This allows quantum computers to process information much faster and more efficiently than classical computers.

COMPARISON BETWEEN CONVENTIONAL COMPUTER AND QUANTUM COMPUTER

Sr. No.	Conventional Computer	Quantum Computer
1	Conventional Computers, no matter how exotic, all follow the law of classical Mechnics/Physics	Quantum computers follow the laws of quantum mechanics and physics.
2	A classical computer has a memory made up of bits	A quantum computer maintains a sequence of qubits
3	A single qubit can represent a one, a zero at a time. It is binary	A single qubit can represent a one, a zero, or a quantum superposition of these.
4	Classical computer can only be in one of these 2n states at any one time	Quantum computer with n qubits can be in an arbitrary superposition of up to 2n different states simultaneously

5.13.2 Concept of Qubit

A qubit, or quantum bit, is the fundamental unit of quantum information in quantum computing. A qubit, unlike a classical bit, can be both 0 and 1 at the same time due to a property known as superposition. Qubits can also be entangled, which means that their states are linked regardless of distance. This enables qubits to interact in ways that classical bits cannot. Because of their unique properties, quantum computers can perform multiple calculations at the same time, making them potentially far more powerful than classical computers for certain applications.

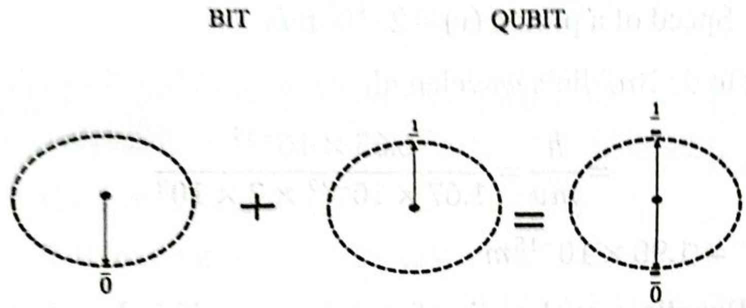

Fig. 5.9: Qubit superposition

5.13.3 Application of Quantum Computing

1. **Artificial Intelligence:** Quantum computing is useful in artificial intelligence.
2. **Molecular Modeling:** Chemical reactions are quantum in nature as they form highly entangled quantum superposition states. Fully-developed quantum computers would evaluate even the most complex processes.
3. **Cryptography:** Quantum computers can accomplish online security by factoring large numbers into primes exponentially more efficiently than digital computers.
4. **Financial Modeling:** Quantum computing can help solve the complicated problems in modern financial market.
5. **Weather Forecasting:** Quantum computer can predict accurate weather forecasting.
6. **Particle Physics:** Quantum computers can help study particle physics that require complex logical operations.

Solved Problems

1. Determine the de-Broglie wavelength of an electron accelerated by a potential difference of 150 V.

Solution: Given: Potential difference (V) = 150 V

Formula: According to de Broglie's wavelength associated with potential difference

$$= \frac{h}{\sqrt{2meV}}$$

$$= \frac{h}{2meV} = \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 150 \times 1.6 \times 10^{-19}}}$$

$$\lambda = 1 \times 10^{-10} \text{m}$$

$$\lambda = 1 \text{ Å}$$

2. A proton is moving with a speed of 2 · 10⁸ m/s. Calculate the wavelength of matter wave associated with it.

Solution: Given Mass of proton (m) = 1.67×10^{-27} kg

Speed of a proton (v) = $2 \cdot 10^{8}$ m/s

Formula: According to de Broglie's wavelength

$$= \frac{h}{mv} = \frac{6.63 \times 10^{-34}}{1.67 \times 10^{-27} \times 2 \times 10^{8}}$$

$$= 1.98 \times 10^{-15} m$$

3. What is the de-Broglie wavelength of an electron, if it has been accelerated by a potential difference of 20 kV?

Solution: Given Potential difference (V) = 20 kV = 20×10^{3} V.

Formula: de - Broglie's wavelength associated with accelerating potential

$$= \frac{h}{\sqrt{2meV}} = \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 1.6 \times 10^{-19} \times 20 \times 10^{3}}}$$

$$= 0.0868 \times 10^{-10}\ m = 0.0868\ A^{0}$$

4. Calculate the de Broglie wavelength of an ⟨- particle accelerated through potential difference of 200 V. ($m_{\alpha} = 6.68 \times 10^{-27} kg$)

Solution: Given $m_{\alpha} = 6.68 \times 10^{-27} kg$ accelerating potential (V) = 200 V

Formula: Charge of an α- particle = 2 × charge on an electron

$= 2 \times 1.6 \times 10^{-19}$ coulomb

de Broglie's wavelength associated with accelerating potential is given by,

$$= \frac{h}{\sqrt{2meV}} = \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 6.68 \times 10^{-27} \times 2 \times 1.6 \times 10^{-19} \times 200}}$$

$$\lambda = 7.175 \times 10^{-13} \text{ m}$$

5. Calculate the velocity and de Broglie wavelength of a neutron of energy 10⁴ eV.

Solution: Given Mass of neutron (m) = 1.67×10^{-27} kg

Energy of neutron (E) = 10^{4} eV = $10^{4} \cdot 1.6 \cdot 10^{-19}$ joules

= $1.6 \cdot 10^{-15}$ joules

Formula: (a) Calculation of velocity of a neutron

$$E = \frac{1}{2}mv^2 \Rightarrow v^2 = \frac{2E}{m}$$

$$\therefore v = \sqrt{\frac{2E}{m}} = \sqrt{\frac{2 \times 1.6 \times 10^{-15}}{1.67 \times 10^{-27}}}$$

$$v = 1.38 \times 10^{6}\ m/s$$

(b) Calculation of de –Broglie wavelength

$$= \frac{h}{mv} = \frac{6.63 \times 10^{-34}}{1.67 \times 10^{-27} \times 1.38 \times 10^{6}}$$

$$= 2.87 \times 10^{-13}\ m$$

6. A neutron of $1.67 \cdot 10^{-27}$ kg mass is moving with a kinetic energy of 20 keV. Calculate the de- Broglie wavelength associate with it.

Solution: Given Mass of a neutron (m) = 1.67×10^{-27} kg

K.E. of neutron (E) = 20 keV = $20 \cdot 10^{3} \cdot 1.6 \cdot 10^{-19}$

$= 3.2 \times 10^{-15}$ joules

Formula: According to de Broglie's wavelength associated with kinetic energy of an electron

$$= \frac{h}{\sqrt{2mE}} = \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 1.67 \times 10^{-27} \times 3.2 \times 10^{-15}}}$$

$$\lambda = 2.071 \times 10^{-13}\ \mathrm{m}$$

7. An electron is accelerated through 1000 volts and is reflected from a crystal. The first order reflection occurs when glancing angle is 70^0. Calculate the interplanar spacing of a crystal.

Solution: Given Potential difference (V) = 1000 Volts

Glancing angle (\) = 70°

Order of reflection (n) = 1

Formula: According to de Broglie's wavelength associated with accelerating potential

$$= \frac{h}{\sqrt{2meV}} = \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 1.6 \times 10^{-19} \times 1000}}$$

$$= 3.88 \times 10^{-11}\ m$$

Bragg's law, $2d\sin\theta = n\lambda$

$$\therefore d = \frac{n}{2\ sin\ sin} = \frac{1 \times 3.88 \times 10^{-11}}{2\ sin\ sin\ 70^0}$$

$$Interplanar\ spacing\ of\ crystal\ (d) = 2.06 \times 10^{-11} m$$

8. Find the K.E. of a neutron which has a wavelength of 3 Å. At what angle will such a neutron undergo first order Bragg reflection from a calcite crystal for which the grating spacing is 3.036 Å.

Solution: Given Wavelength = 3 Å = 3×10^{-10} m

Order of Bragg reflection (n) = 1

Interplanar spacing (d) = 3.036 Å = 3.036×10^{-10} m

Formula: (a) Calculation of kinetic energy of a neutron

de - Broglie's wavelength associated with kinetic energy is given by,

$$= \frac{h}{\sqrt{2mE}} \Rightarrow \quad ^2 = \frac{h^2}{2mE}$$

$$\therefore E = \frac{h^2}{2m \quad ^2} = \frac{(6.63 \times 10^{-34})^2}{2 \times 1.67 \times 10^{-27} \times (3 \times 10^{-10})^2}$$

$$Kinetic\ energy\ of\ a\ neutron\ (E) = 1.46 \times 10^{-21}\ joules$$

(b) Calculation of Bragg's angle of reflection for first order reflection

Bragg's law, 2 d sin θ = nλ

$$\therefore sin\ sin \ = \frac{n}{2d}$$

$$sin\ sin \ = \frac{1 \times (3 \times 10^{-10})}{2 \times (3.036 \times 10^{-10})} = 0.4940$$

$$Bragg's\ angle\ of\ reflection\ (\theta) = sin^{-1}(0.4940) = 30^o$$

9. The electrons which are at rest accelerated through a potential difference of 250 V. Calculate

(a) The velocity of an electron **(b) Phase velocity of an electron**

(c) de- Broglie's wavelength **(d) Momentum and**

(e) Wave number of an electron wave.

Solution: Given Potential difference (V) = 250V

Formula: (a) The velocity of an electron

Kinetic energy = Potential energy.

$$\frac{1}{2}mv^2 = eV$$

$$\therefore v = \sqrt{\frac{2eV}{m}} = \sqrt{\frac{2 \times 1.6 \times 10^{-19} \times 250}{9.1 \times 10^{-31}}}$$

The velocity of an electron (v) = 9.376×10^6 m/s

(b) Phase velocity of an electron

$$u = \frac{c^2}{v} = \frac{(3 \times 10^8)^2}{9.376 \times 10^6}$$

Phase velocity of an electron (u) = 9.59×10^9 m/s

(c) de- Broglie's wavelength

$$= \frac{h}{mv} = \frac{6.63 \times 10^{-34}}{9.1 \times 10^{-31} \times 9.376 \times 10^{6}}$$

de- Broglie's wavelength $(\lambda) = 7.78 \times 10^{-11}$ *m*

(d) Momentum

$$p = m\,v$$

$$= 9.1 \times 10^{-31} \times 9.376 \times 10^{6}$$

Momentum (p) $= 8.532 \times 10^{-24}$ *kg m/sec.*

(e) Wave number of an electron wave.

$$\underline{v} = \frac{1}{\lambda} = \frac{1}{7.78 \times 10^{-11}}$$

Wave number of an electron wave $= 1.28 \times 10^{10}$ */m*

10. If the uncertainty in position of an electron is $4 \cdot 10^{-10}$ m, calculate the uncertainty in its momentum.

Solution: Given Uncertainty in measurement of position $(\Delta x) = 4 \times 10^{-10}$ m

Formula: According to Heisenberg's uncertainly principle,

$$\Delta x . \Delta p \geq \hbar$$

$$\Delta x . \Delta p \simeq \hbar$$

$$\Delta x . \Delta p = \frac{h}{2\pi}$$

$$\Delta p = \frac{h}{2\pi} \times \frac{1}{\Delta x} = \frac{6.63 \times 10^{-34}}{2\pi} \times \frac{1}{4 \times 10^{-10}}$$

Uncertainty in measurement of momentum (Δp) = 2.63×10^{-25} kg-m/sec

11. An electron has a speed of 300 m/s with an accuracy of 0.001%. Calculate the certainty with which we can locate the position of an electron.

Solution: Given Speed of an electron (v) = 300 m/s

$$(\Delta v) = \frac{300 \times 0.001}{100} = 0.003\ m/s$$

Formula: According to Heisenberg's uncertainty principle,

$$\Delta x . \Delta p \geq \hbar$$

$$\Delta x . \Delta p \simeq \hbar$$

$$\Delta x . \Delta p = \frac{h}{2\pi}$$

$$\Delta x = \frac{h}{2\pi} \times \frac{1}{\Delta p}$$

$$\Delta x = \frac{h}{2\pi} \times \frac{1}{m\,\Delta v} = \frac{6.63 \times 10^{-34}}{2\pi} \times \frac{1}{9.1 \times 10^{-31} \times 0.003}$$

$$Certainty\ in\ location\ of\ the\ position\ (\Delta x) = 0.0386\ m$$

12. A bullet of mass 0.050 kg is moving with a velocity of 800 m/s the speed of a bullet is measured accuracy 0.01%. Calculate the accuracy with which the position of an bullet can be located.

Solution: Given Mass of bullet (m) = 0.050 kg

Velocity of bullet (v) = 800 m/s

$$(\Delta v) = \frac{800 \times 0.01}{100} = 0.08\ m/s$$

Formula: According to Heisenberg's uncertainty principle,

$$\Delta x . \Delta p \geq \hbar$$

$$\Delta x . \Delta p \simeq \hbar$$

$$\Delta x . \Delta p = \frac{h}{2\pi}$$

$$\Delta x = \frac{h}{2\pi} \times \frac{1}{\Delta p}$$

$$\Delta x = \frac{h}{2\pi} \times \frac{1}{m\,\Delta v} = \frac{6.63 \times 10^{-34}}{2\pi} \times \frac{1}{0.050 \times 0.08}$$

$$\text{Accuracy in location of position } (\Delta x) = 2.62 \times 10^{-32}\ m$$

13. If the uncertainty in position of an electron is $4 \cdot 10^{-10}$ m. Calculate the uncertainty in its momentum.

Solution: Given: Uncertainty in position of an electron $(\Delta x) = 4 \times 10^{-10}$ m

Formula: According to Heisenberg's uncertainty principle,

$$\Delta x . \Delta p \geq \hbar$$

$$\Delta x . \Delta p \simeq \hbar$$

$$(\Delta x).(\Delta p) = \frac{h}{2\pi}$$

$$(\Delta p) = \frac{h}{2\pi} \times \frac{1}{(\Delta x)} = \frac{6.63 \times 10^{-34}}{2\pi \times 4 \times 10^{-10}}$$

$$(\Delta p) = 2.637 \times 10^{-25}\ kg\ m/sec$$

14. If diameter of the nucleus is 8.5 ·10^{-14} m. Calculate minimum momentum of a proton and minimum kinetic energy of proton.

Solution: Given: Diameter of the nucleus $(\Delta x) = 8.5 \cdot 10^{-14}$ m

Formula: (a) Calculation of minimum momentum of a proton

According to Heisenberg's uncertainty principle,

$$\Delta x.\Delta p \geq \hbar$$

$$\Delta x.\Delta p \simeq \hbar$$

$$(\Delta x)_{max}.(\Delta p)_{min} = \frac{h}{2\pi}$$

$$(\Delta p)_{min} = \frac{h}{2\pi} \times \frac{1}{(\Delta x)_{max}} = \frac{6.63 \times 10^{-34}}{2\pi} \times \frac{1}{8.5 \times 10^{-14}}$$

$$Minimum\ momentum\ (p_{min}) = (\Delta p)_{min} = 1.241\ \times 10^{-21}\ kg\ m/s$$

(b) Calculation of minimum energy of proton

$$E = \frac{mv^2}{2} = \frac{m^2v^2}{2m}$$

$$(E)_{min} = \frac{(p^2)_{min}}{2m} = \frac{(1.241 \times 10^{-21})^2}{2 \times 1.67 \times 10^{-27}}$$

$$(E)_{min} = 4.611 \times 10^{-16}\ joules$$

$$(E)_{min} = \frac{1.150 \times 10^{-16}}{1.6 \times 10^{-19}}$$

$$Minimum\ energy\ of\ proton\ in\ (E)_{min} = 2881\ eV$$

15. A hydrogen atom has diameter of 0.53 Å. Estimate the minimum energy an electron can have in this atom.

Solution: Given $(\Delta x) = 0.53$ Å $= 0.53 \times 10^{-10}$ m

Formula: (a) Calculation of minimum momentum of an electron

According to Heisenberg's uncertainty principle,

$$\Delta x.\Delta p \geq \hbar$$

$$\Delta x.\Delta p \simeq \hbar$$

$$(\Delta x)_{max}.(\Delta p)_{min} = \frac{h}{2\pi}$$

$$(\Delta p)_{min} = \frac{h}{2\pi} \times \frac{1}{(\Delta x)_{max}}$$

$$(\Delta p)_{min} = \frac{h}{2\pi} \times \frac{1}{(\Delta x)_{max}} = \frac{6.63 \times 10^{-34}}{2\pi} \times \frac{1}{0.53 \times 10^{-10}}$$

$$(\Delta p)_{min} = 1.990 \times 10^{-24}\ kg\ m/s$$

(b) Calculation of minimum of energy of an electron

$$Kinetic\ energy = \frac{1}{2} mv^2 = \frac{m^2v^2}{2m} = \frac{p^2}{2m}$$

$$Kinetic\ energy\ (E) = \frac{p^2}{2m}$$

$$Kinetic\ energy (E)_{min} = \frac{(p^2)_{min}}{2m} \qquad [(p)_{min} = (\Delta p)_{min}]$$

$$= \frac{(1.990 \times 10^{-24})^2}{2 \times 9.1 \times 10^{-31}}$$

Minimum of energy of an electron = 2.175 × 10^{-18} joules

16. An electron confined in a box of 10^{-7} m length. Calculate minimum uncertainty in its velocity.

Solution: **Given** $(\Delta x) = 10^{-7}$ m

Formula: According to Heisenberg's uncertainty principle,

$$\Delta x . \Delta p \geq \hbar$$

$$\Delta x . \Delta p \simeq \hbar$$

$$(\Delta x)_{max} \cdot (\Delta p)_{min} = \frac{h}{2\pi}$$

$$(\Delta p)_{min} = \frac{h}{2\pi} \times \frac{1}{\Delta x} = \frac{6.63 \times 10^{-34}}{2\pi} \times \frac{1}{10^{-7}}$$

$$(\Delta p)_{min} = 1.055 \times 10^{-27}\ \text{kg-m/sec}$$

$$(\Delta p)_{min} = m\ (\Delta v)_{min}$$

$$(\Delta v)_{min} = \frac{(\Delta p)_{min}}{m} = \frac{1.055 \times 10^{-27}}{9.1 \times 10^{-31}}$$

$$(\Delta v)_{min} = 1159\ m/sec$$

17. Find out the lowest energy of an electron in a one dimensional box width of 4Å.

Solution: **Given** Width of box (a) = 4 Å = 4 ·10^{-10} m

Formula: Energy of an electron in one dimensional potential well

$$E_n = \frac{n^2h^2}{8ma^2}$$

The lowest energy of an electron can be calculated by putting n =1

$$E_1 = \frac{1^2 \times (6.63 \times 10^{-34})^2}{8 \times 9.1 \times 10^{-31} \times (4 \times 10^{-10})^2} = 3.7731 \times 10^{-19}\ joules$$

$$E_1 = \frac{3.7731 \times 10^{-19}}{1.6 \times 10^{-19}}$$

The lowest energy an electron in electron volt = 2.358 eV

18. Calculate minimum energy that an electron posses in an infinity deep potential well of 2 nm width.

Solution: Given: Width of the potential well (a) = 2 nm = 2 $\times 10^{-9}$ m

Formula: Energy of an electron in one dimensional potential well

$$E = \frac{n^2h^2}{8ma^2}$$

Minimum energy possessed by an electron can be calculated by putting n =1

$$E = \frac{1^2 \times (6.63 \times 10^{-34})^2}{8 \times 9.1 \times 10^{-31} \times (2 \times 10^{-9})^2}$$

$$E = 1.5 \times 10^{-20}\ joules$$

$$E = \frac{1.5 \times 10^{-20}}{1.6 \times 10^{-19}} = 0.0943\ eV$$

Energy of an electron in electron volts = 0.0943 eV

19. Find the energy of an electron moving in one dimension in an infinitely high potential box of width 1 Å. Given: mass of an electron is 9.1 $\cdot 10^{-31}$ kg and h = 6.63 $\cdot 10^{-34}$ J-s.

Solution: Given: Width of a box (a) = 1 Å = 10^{-10} m

Mass of an electron (m) = 9.1 $\cdot 10^{-31}$ kg

Planck's constant (h) = 6.63 $\cdot 10^{-34}$ J-s.

Formula: Energy of an electron in one dimensional potential well

$$E = \frac{h^2}{8ma^2} = \frac{(6.63 \times 10^{-34})^2}{8 \times 9.1 \times 10^{-31} \times (10^{-10})^2}$$

$$E = 6.03 \times 10^{-18}\ joules$$

$$E = \frac{6.03 \times 10^{-18}}{1.6 \times 10^{-19}}$$

Energy of an electron in one dimensional potential well = 37.73 eV

20. An electron has a speed of 900 m/s with an accuracy of 0.001%. Calculate the uncertainty with which the position of the electron can be located.

Solution: **Given** Speed of an electron (v) = 900 m/s

Formula: Accuracy in measurement of speed of an electron

$$(\Delta v) = \frac{0.001}{100} \times 900 = 9 \times 10^{-3}\ m/s$$

According to Heisenberg's uncertainty principle,

$$\Delta x. \Delta p \geq \hbar$$

$$\Delta x. \Delta p \simeq \hbar$$

$$\Delta x. \Delta p = \frac{h}{2\pi}$$

$$\Delta x. m\Delta v = \frac{h}{2\pi}$$

$$\Delta x = \frac{h}{2\pi} \times \frac{1}{m\Delta v} = \frac{6.63 \times 10^{-34}}{2\pi} \times \frac{1}{9.1 \times 10^{-31} \times 9 \times 10^{-3}}$$

$$Uncertainty\ in\ location\ of\ position\ (\Delta x) = 0.01288\ m$$

21. Find the energy of the neutron in units of electron volt whose de-Broglie wavelength is 1 Å (Given $m_n = 1.67 \times 10^{-27}$ kg, h = 6.63×10^{-34} J-sec.)

Solution: **Given** de- Broglie wavelength (λ) = 1 Å = 1×10^{-10} m

Mass of the neutron = $1.67 \cdot 10^{-27}$ kg

Planck's constant (h) = $6.63 \cdot 10^{-34}$ J-sec.

Formula: de Broglie wavelength

$$\lambda = \frac{h}{\sqrt{2mE}}$$

$$E = \frac{h^2}{2m\lambda^2} = \frac{(6.63 \times 10^{-34})^2}{2 \times 1.67 \times 10^{-27} \times (1 \times 10^{-10})^2}$$

E = 1.31×10^{-20} J

$$E = \frac{1.31 \times 10^{-20}}{1.6 \times 10^{-19}}\ eV \qquad (1eV = 1.6 \times 10^{-19}\ Joule)$$

Energy of the neutron in units of electron volt (E) = 0.082 eV

22. An electron has a speed of 400 m/sec with uncertainly of 0.01%. Find the accuracy in its position.

Solution: **Given** Speed of an electron (v) = 400 m/s.

Uncertainty in measurement of velocity $(\Delta v) = \frac{400 \times 0.01}{100} = 0.04\ m/s$

Formula: (a) Calculation of Δp

$$\Delta p = m.\ \Delta v = 9.1\ 10^{-31} \times 0.04$$

$$\mathbf{\Delta p = 3.64 \times 10^{-32}\ kg\text{-}\ m/sec.}$$

According to Heisenberg's uncertainty principle,

$$\Delta x. \Delta p \geq \hbar$$

$$\Delta x. \Delta p \simeq \hbar$$

$$\Delta x. \Delta p = \frac{h}{2\pi}$$

$$\Delta x = \frac{h}{2\pi} \times \frac{1}{\Delta p} = \frac{6.63 \times 10^{-34}}{2\pi} \times \frac{1}{3.64\ 10^{-32}}$$

$$Accuracy\ in\ measurement\ of\ \ position\ (\Delta x) = 2.898\ \times 10^{-3}\ m$$

23. What is the wavelength of a beam of neutrons having energy 0.025 eV and mass 1.676 · 10^{-27} kg?

Solution: Given Energy of neutron (E) = 0.025 eV = 0.025 × 1.6 × 10^{-19} J

Mass of a neutron (m) = 1.676 · 10^{-27} kg

Formula: We have, $E = \frac{1}{2}mv^2$

$$v = \sqrt{\frac{2E}{m}} = \sqrt{\frac{2 \times 0.025\ 1.6\ \ 10^{-19}}{1.676 \times 10^{-27}}}$$

$$\mathbf{v = 2.188 \times 10^3\ m/s}$$

According to de Broglie's wavelength

$$= \frac{h}{mv} = \frac{6.63 \times 10^{-34}}{1.67 \times 10^{-27} \times 2.188 \times 10^3}$$

$$\mathbf{= 1.814 \times 10^{-10}\ m}$$

$$\mathbf{\lambda = 1.814\ Å}$$

24. The position and momentum of 1 keV electron are simultaneously measured. If its position is located within 10 nm, then what is the percentage of uncertainty in its momentum.

Solution: Given Energy of electron (E) = 1Kev = 1 × 10^3 eV =10^3 × 1.6 × 10^{-19} J

Uncertainty in measurement of position (Δx) = 10 nm = 10 ·10^{-9} m

Formula: $K.E. = \frac{1}{2}mv^2$

$$E = \frac{1}{2}\frac{m^2v^2}{m} = \frac{p^2}{2m}$$

$$p^2 = 2mE$$

$$p = \sqrt{2mE} = \sqrt{2 \times 9.1 \times 10^{-31} \times 10^3 \times 1.6 \times 10^{-19}}$$

p = 1.706 ×10^{-23} kg m/sec

According to Heisenberg's uncertainly principle,

$$\Delta x . \Delta p \geq \hbar$$

$$\Delta x . \Delta p \simeq \hbar$$

$$\Delta x . \Delta p = \frac{h}{2\pi}$$

$$\Delta p = \frac{h}{2\pi} \times \frac{1}{\Delta x}$$

$$\Delta p = \frac{6.63 \times 10^{-34}}{2\pi} \times \frac{1}{10 \times 10^{-9}}$$

Δp = 1.055 ×10^{-26} kg.m/sec

Percentage of uncertainty in measurement of momentum

$$\frac{\Delta p}{p} \times 100 = \frac{1.055 \times 10^{-26}}{1.7062 \times 10^{-23}} \times 100$$

$$= 6.18 \times 10^{-4}\ \%$$

25. An electron is bound in a one-dimensional potential well of width 2 Å, but of infinite height. Find its energy values in the ground state and first two excited states.

Solution: Given Width of potential well (a) = 2 Å = 2 × 10^{-10} m

Formula: Energy of an electron in one dimensional potential well

$$E_n = \frac{n^2 h^2}{8ma^2}$$

(a) Calculation of energy of an electron in the ground state [Put n =1]

$$E_n = \frac{1^2 \times (6.63 \times 10^{-34})^2}{8 \times 9.1 \times 10^{-31} \times (2 \times 10^{-10})^2} = 1.5 \times 10^{-18}\, J$$

$$E_1 = \frac{1.5 \times 10^{-18}}{1.6 \times 10^{-19}} \qquad (1eV = 1.6 \times 10^{-19}\, joule)$$

Energy of an electron in the first excited state (E$_1$) = 9.375 eV

(b) Calculation of energy of an electron in the first excited state [Put, n = 2]

$$E_2 = \frac{2^2 \times (6.63 \times 10^{-34})^2}{8 \times 9.1 \times 10^{-31} \times (2 \times 10^{-10})^2} = 6 \times 10^{-18}\, J$$

$$E_2 = \frac{6 \times 10^{-18}}{1.6 \times 10^{-19}} = 37.5\ eV$$

Energy of an electron in the first excited state (E_2) = 37.5 eV

(c) Calculation of energy of an electron in the second excited state [Put, n = 3]

$$E_3 = \frac{3^2 \times (6.63 \times 10^{-34})^2}{8 \times 9.1 \times 10^{-31} \times (2 \times 10^{-10})^2} = 1.35 \times 10^{-17}\, J$$

$$E_3 = \frac{1.35 \times 10^{-17}}{1.6 \times 10^{-19}} = 84.375\, eV$$

Energy of an electron in the second excited state (E_3) = 84.375 eV

26. Calculate the de- Broglie wavelength of an electron whose kinetic energy is 120 eV. [$m = 9.1 \times 10^{-31}$ kg, $h = 6.63 \times 10^{-34}$ J-s].

Solution: Given Mass of an electron = 9.1×10^{-31} kg

Planck's constant (h) = $6.63 \cdot 10^{-34}$ J.s

Kinetic energy of an electron (E) =120 eV = $120 \cdot 1.6 \cdot 10^{-19}$ J

Formula: de Broglie's wavelength associated with kinetic energy is given by,

$$= \frac{h}{\sqrt{2mE}}$$

$$= \frac{h}{\sqrt{2mE}} = \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 120 \times 1.6 \times 10^{-19}}}$$

$$\lambda = 1.12 \times 10^{-10}\ \text{m} = 1.12\ \text{Å}$$

27. An electron moves in the x-direction with a speed of 1.88×10^6 m/s. if this speed is measured to a precision of 1%, with what precision can you simultaneously measure its position?

Solution: Given Velocity of an electron (v) = 1.88×10^6 m/sec

Formula: Uncertainty in measurement of speed

$$(\Delta v) = \frac{1.88 \times 10^6}{100} = 1.88 \times 10^4\ m/s$$

According to Heisenberg's uncertainly principle,

$$\Delta x . \Delta p \geq \hbar$$

$$\Delta x . \Delta p \simeq \hbar$$

$$\Delta x . \Delta p = \frac{h}{2\pi}$$

$$\Delta x = \frac{h}{2\pi} \times \frac{1}{\Delta p}$$

$$\Delta x = \frac{h}{2\pi} \times \frac{1}{m \Delta v}$$

$$\Delta x = \frac{6.63 \times 10^{-34}}{2\pi} \times \frac{1}{9.1 \times 10^{-31} \times 1.88 \times 10^{4}}$$

Accuracy in measurement of position (Δx) = 6.16 × 10^{-9} m = 6.16 nm

28. Calculate the de-Broglie wavelength of proton traveling with a velocity equal to 1/20th velocity of light. (Mass of proton = 1.67 · 10^{-27} kg.

Solution: **Given** Velocity of proton (v) = $\frac{1}{20} \times 3 \times 10^{8}\ m/s$

Mass of proton (m_p) = $1.67 \cdot 10^{-27}$ kg

Formula: de- Broglie wavelength

$$= \frac{h}{mv} = \frac{6.63 \times 10^{-34}}{1.67 \times 10^{-27} \times \frac{1}{20} \times 3 \times 10^{8}}$$

de- Broglie wavelength (λ) = 2.7 × 10^{-14} m.

SHORT ANSWER TYPE QUESTIONS

1. Explain wave particle duality of radiation.
2. What are the properties of a photon?
3. State de-Broglie hypothesis of matter waves.
4. What are properties of matter waves?
5. Obtain an expression for de-Broglie wavelength associated with an electron accelerated through a potential V.
6. Derive an expression for de-Broglie wavelength associated with a particle moving with kinetic energy K.E.
7. Show that the de-Broglie's wavelength of a charged particle is inversely proportional to square root of potential difference.
8. What is the physical significance of the wave function Ψ of a matter wave?
9. Define a wave function. What do you mean by normalized wave function?
10. Explain concept of a wave packet.
11. Explain phase velocity of a wave group and group velocity of matter waves.
12. Explain group and phase velocity.
13. State and explain Heisenberg's uncertainty principle
14. Explain the concept of quantum computing. Write applications of quantum computing
15. Differentiate between classical and quantum computing

DESCRIPTIVE ANSWER TYPE QUESTIONS

1. State De-Broglie's hypothesis. Derive expression for De Broglie's wavelength.
2. Explain de Broglie model of an atom. Obtain de Broglie wavelength of orbiting electron and circumference of electron orbit in hydrogen atom. (Radius of electron orbit is 5.3×10^{-11} m).

3. Using Heisenberg's uncertainty principle, prove that electron cannot pre-exist in atomic nucleus.
4. What do you understand by a wave packet? Obtain an expression for the Bohr's condition for quantization of angular momentum.
5. Explain phase velocity and group velocity. Derive the relation between them
6. Explain Heisenberg's uncertainly principle with an example and give its physical significance.
7. Prove the non-existence an electron and existence proton exists within the nucleus using uncertainty principle
8. What is uncertainty principle? Explain how the concept of wave particle duality and single slit diffraction of electron can be used to prove the uncertainty principle?
9. State Heisenberg's uncertainty principle. Describe experimental verification of uncertainty principle.
10. What is de-Broglie concept of matter-waves? Derive one dimensional time-dependent Schrodinger equation for matter waves.
11. Drive the one-dimensional time independent Schrodinger wave equation for matter waves.
12. Discuss the energy levels of an electron enclosed in one dimensional infinite potential well.
13. What is quantum computing? What makes a quantum computer different from a regular computer?

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11. Drive the one-dimensional time independent Schrodinger wave equation for matter waves.
12. Discuss the energy levels of an electron enclosed in one dimensional infinite potential well.
13. What is quantum computing? What makes a quantum computer different from a regular computer?

CHAPTER 6.

Basics of Semiconductor Physics

6.1 INTRODUCTION

In the modern world, technology is all around us, changing the way we live, work, and communicate. At the heart of this technological revolution are semiconductors, materials that have revolutionized the way we live, work, and communicate. From the smartphones in our pockets to the laptops on our desks, from medical equipment to spacecraft, semiconductors play a vital role in enabling the functionality of modern electronics. The understanding of electron behavior in solids, which began with the discovery of the electron in 1897, has led to the development of semiconductor technology, transforming the world as we know it. Today, semiconductors are a crucial component of modern technology, driving innovation and progress in fields like computing, healthcare, energy, and transportation. As we continue to push the boundaries of what is possible, the importance of semiconductors will only continue to grow, making the study of electron behavior in solids an essential foundation for the technologies of tomorrow.

6.1.1 FORMATION OF ENERGY BANDS IN SOLIDS

In solids, the close proximity of atoms leads to the formation of energy bands. This phenomenon occurs due to the interaction of valence electrons, resulting in new energy levels and the smearing out of discrete levels into bands. As atoms come together, their energy levels split into multiple levels as shown in (Fig 6.1), a process known as level splitting, resulting in a large number of closely spaced energy levels.

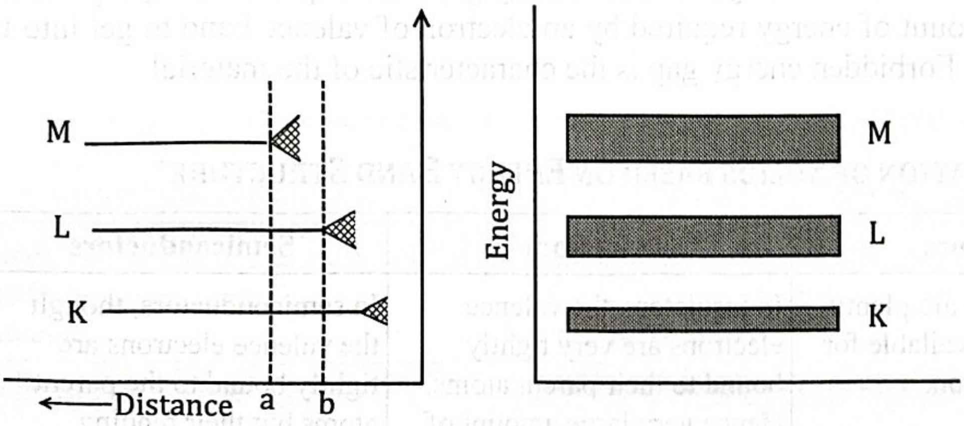


Fig. 6.1 Energy bands in solids

Fig (6.1) shows the splitting of energy levels K, L and M when distance between the atoms is gradually decreased. M level being the outermost i.e. valance level is affected first say at a distance ***a***. Subsequently L level undergoes splitting as the interatomic

distance is reduced further say at ***b***. The energy bands formed by modification of these levels are ***wider energy bands.*** However the electrons in the innermost orbits are ***TIGHTLY BOUND TO THEIR NUCLEI DUE TO THE FORCE OF ATTRACTION AND THEREFORE THE INNERMOST*** energy levels are less affected by the presence of the neighbouring atoms. As a result they form ***narrow energy bands.***

6.1.2 VALENCE BAND, CONDUCTION BAND, FORBIDDEN ENERGY GAP

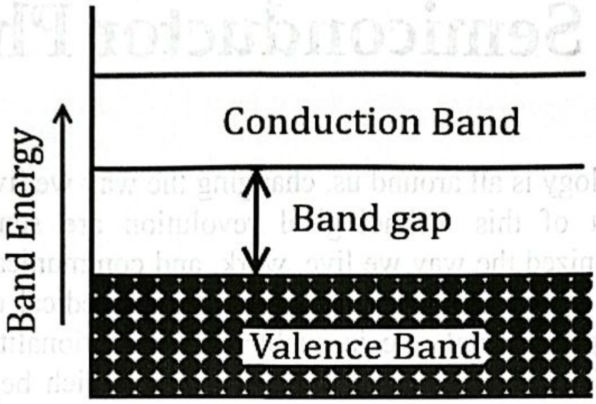


Fig. 6.2 Valence band, Conduction band and Forbidden gap

Valence band

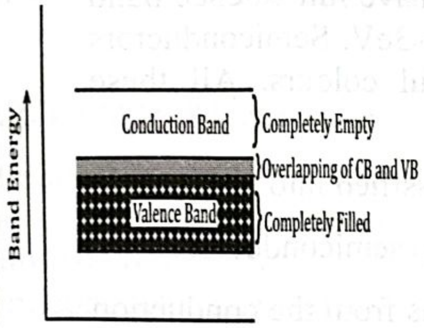
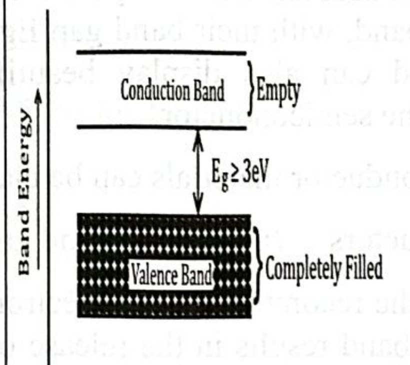
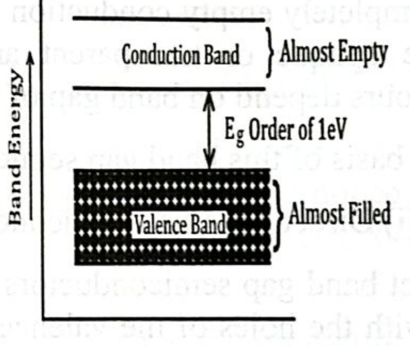
The band of series of energy levels occupied by valence electrons is known as valence band. It is the highest occupied band. It may be *completely or partially filled* with electrons. The valence band is responsible for electrical, thermal and optical properties of solids.

Conduction band: The next higher permitted energy band is known as ***conduction band***. It may be *empty or partially filled*. In this band electrons move freely and contribute in the electrical conduction hence these free electrons are known as ***conduction electrons.***

Forbidden energy gap: The gap between top edge of valence and the bottom edge of the conduction band is known as ***forbidden energy gap.*** It is represented by E_g and it is the minimum amount of energy required by an electron of valence band to get into the conduction band. Forbidden energy gap is the characteristic of the material.

6.1.3 CLASSIFICATION OF SOLIDS BASED ON ENERGY BAND STRUCTURE

Conductors	Insulators	Semiconductors
In conductor there are plenty of free electrons available for electrical conduction.	In insulators, the valence electrons are very tightly bound to their parent atoms. Hence very large amount of energy is required to remove them from the attraction of nuclei.	In semiconductors, though the valence electrons are tightly bound to the parent atoms but their binding energy is comparatively less to that of insulators.

In terms of energy band theory, the conduction and valence band overlap each other.	In terms of energy band theory, the insulators have completely filled valence band, an empty conduction band and a very large forbidden energy gap (E_g). (For insulators $E_g > 3$ eV)	In terms of energy band theory the insulators have almost empty conduction band, almost filled valence band and a small forbidden energy gap E_g of the order of 1eV.
The electrical resistance of a conductor increases with increasing temperature i.e. they have positive temperature coefficient of resistance.	The electrical resistance of an insulator decreases with increasing temperature i.e. they have negative temperature coefficient of resistance.	The electrical resistance of a semiconductor decreases with increasing temperature i.e .they have negative temperature coefficient of resistance.
The conductivity of conductors is of the order of 10^7 mho/meter at room temperature.	The conductivity of insulators is of the order of 10^{-13} mho/meter at room temperature.	The conductivity of semiconductors lies between 10^{-4} to 10^4 mho/meter at room temperature.
Example: copper, silver, iron, aluminum, tin, lead, mercury etc.	Example: diamond ($E_g \approx 6$ eV) and for glass ($E_g \approx$ 10eV).	Example: Semiconductors are germanium (E_g=0.72 eV) and silicon (E_g=1.12 eV).
		

6.1.4 DIFFERENCE BETWEEN INTRINSIC AND EXTRINSIC SEMICONDUCTORS

Intrinsic Semiconductors	Extrinsic Semiconductors
The semiconductor which is of extremely pure form is known as *intrinsic semiconductors.*	The intrinsic semiconductors doped with impurity atoms are known as extrinsic semiconductors.
They have low electrical conductivity.	They have comparatively higher electrical conductivity.

The electrical conductivity of these semi-conductors depends upon temperature.	The electrical conductivity of these semi-conductors depends upon the temperature as well as the concentration of doped impurity atoms.
The concentration of free electrons in the conduction band and holes in valence band are equal.	The concentration of free electrons in the conduction band and holes in valence band are different. It depends upon type of impurity atoms added to it.
The Fermi-energy level lies in between energy of valence and conduction band.	The position of Fermi-energy level depends upon impurity concentration and temperature.
Comparatively more amount of energy is required for electrons to cross the forbidden energy gap.	Comparatively less amount of energy is required for electrons to cross the forbidden energy gap as compared to intrinsic semiconductor.
Examples: Germanium (0.72 eV) and Silicon (1.12 eV).	Examples: silicon and germanium crystals doped with impurity atoms of As, Sb, P, In, B etc.

6.2 Direct and Indirect Band Gap Semiconductors

We can recall that semiconductors at absolute zero temperature have full valence band and completely empty conduction band, with their band gap Eg<3eV. Semiconductors can be opaque or transparent and can also display beautiful colours. All these behaviours depend on band gap of the semiconductor.

On the basis of this band gap semiconductor materials can be classified into two types:

(i) Direct band gap semiconductors (ii) Indirect band gap semiconductors

In direct band gap semiconductors the recombination of electrons from the conduction band with the holes of the valence band results in the release of photon in the optical region of the electromagnetic spectrum. Examples are GaAs, GaAsP, InAs, ZnS, etc. These semiconductors have high optoelectronic conversion efficiency and hence widely used in optoelectronic devices such as LED, Lasers, etc. The optical properties of these compounds can be very effectively controlled by controlling their composition and thus managing the band gap.

When the recombination takes place between an electron in energy level E_2 and a hole in energy level E_1. The energy of the photon emitted is given by $E_2 - E_1$. If the wavelength of the emitted photon lies in the visible range, then a photon is said to be emitted.

The energy of emitted photon is given by, $E_2 - E_1 = h\upsilon$ (6.1)

$$E_2 - E_1 = hc/\lambda \quad (6.2)$$

Where, h is the Planck's constant, υ is the frequency, c is the velocity and λ is the wavelength. The numerator in R.H.S. of eq (6.2) is a constant. Thus one can conclude

that energy of released in form of recombination is inversely proportional to the wavelength of the emitted photon.

In indirect band gap semiconductors, the recombination of the electrons and holes require lattice defects as recombination centers resulting in lattice vibration. This results in the release of energy in the form of heat and not in form of light. Examples: Si, Ge, GaP, α-SiC, etc.

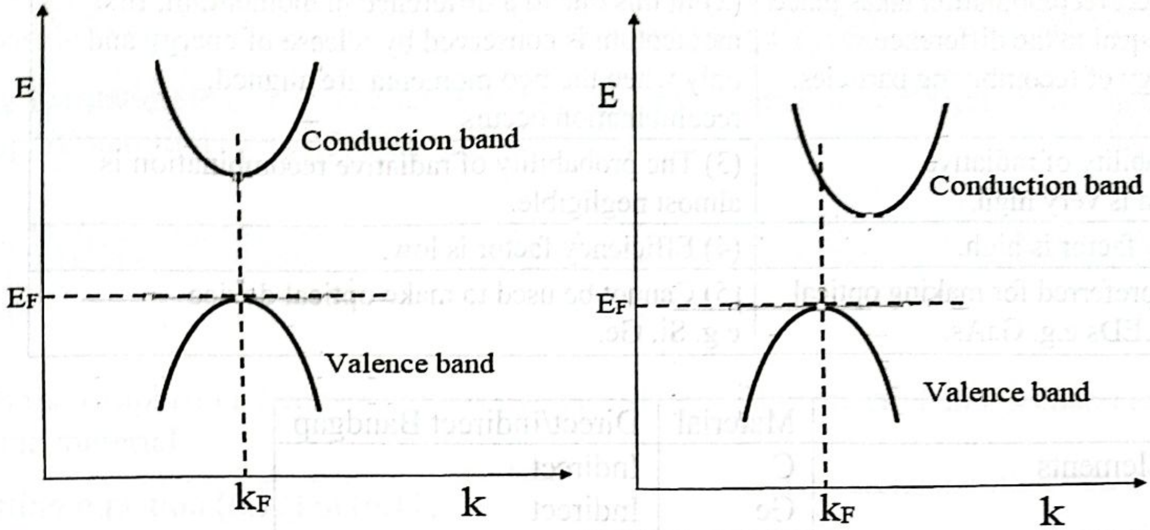


Fig. 6.3

(i) Direct band gap semiconductors (ii) Indirect band gap semiconductors

6.2.1 ELEMENTAL AND COMPOUND SEMICONDUCTORS

The most commonly known semiconductors are Silicon and Germanium belonging to the group IV of the periodic table. Commercially these are used in pure and doped (intrinsic or extrinsic) form. They are known as elemental semiconductors. This type of semiconductor mainly have indirect band gap.

Semiconductors can also be formulated by combining different elements from two or more different groups of the periodic table: III-IV, II-VI, and IV-VI. These types of semiconductors are known as compound semiconductors, majority of them having direct band gap emitting photon in the optical region. These compound semiconductors open up a huge collection of compounds. They can be classified into three groups, namely:

(i) Binary Compounds (ii) Ternary Compounds (iii) Quaternary Compounds

As the name suggests binary compounds are formed by combination of elements from two group of the periodic table. Some examples of compounds belonging to group III-IV are GaAs, InP, GaP, AlSb and group II-VI are ZnS, CdS, ZnSe. Similarly ternary compounds are formed by combining three elements and quaternary compounds by grouping four elements from the different groups of the periodic table. Example of ternary compound is InGaAs and quaternary compound is AlInGaP.

6.2.2 DIFFERENTIATE BETWEEN DIRECT AND INDIRECT BAND GAP SEMICONDUCTORS

Direct Band Gap (DBG) semiconductor	Indirect Band Gap (IBG) semiconductor
(1) It is one in which maximum energy level of valence band aligns with the minimum energy level of conduction band with respect to momentum.	(1) It is the one in which maximum energy level of valence band and minimum energy level of valence band and minimum energy level of conduction band are not aligned with respect to momentum.
(2) In this direct recombination takes place with energy equal to the difference between energy of recombining particles.	(2) In this due to a difference in momentum, first momentum is conserved by release of energy and only when the two momenta are aligned, recombination occurs.
(3) The probability of radiative recombination is very high.	(3) The probability of radiative recombination is almost negligible.
(4) Efficiency factor is high.	(4) Efficiency factor is low.
(5) They are preferred for making optical devices like LEDs e.g. GaAs.	(5) Cannot be used to make optical device e.g. Si, Ge.

	Material	Direct/Indirect Bandgap
Elements	C Ge Si	Indirect Indirect Indirect
Group III-V Compound	GaAs InAs GaP GaN	Direct Direct Indirect Direct
Group II-VI Compound	ZnO CdSe ZnS	Direct Direct Direct

6.3 ELECTRICAL CONDUCTION

6.3.1 DRIFT VELOCITY, MOBILITY AND CONDUCTIVITY IN CONDUCTORS

According to free electron theory of metals, a metal piece consists of plenty of free electrons available for conduction of electricity. Let us consider a conductor of cross sectional area *A*, length *l* and *V* be potential difference applied across it.

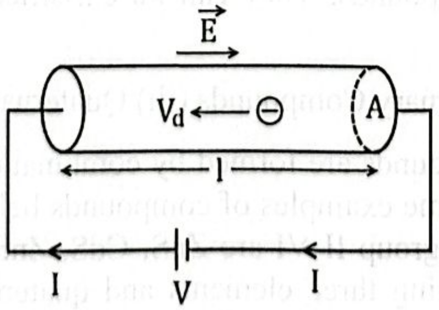


Fig. 6.4 Electrical Conductivity and Resistivity of conductors

An electric field *E* established due to the potential difference *V* in the conductor is given by

$$E = \frac{V}{l}$$

Ohm's law that governs the electrical conduction in metals is

$$V = IR$$

$$I = \frac{V}{R} \tag{6.11}$$

where V is the potential difference, I the current and R the resistance of the conductor.

The resistance R of the conductor is directly proportional to its length l and inversely proportional to its area of cross-section A. Thus,

$$R \propto \frac{l}{A}$$

$$R = \rho \frac{l}{A} \tag{6.12}$$

ρ is the proportional constant known as **resistivity** of the material and is characteristic of the material.

Putting equation (6.12) in (6.11)

$$I = \frac{VA}{l\rho} \tag{6.13}$$

The conductivity (σ) is reciprocal of resistivity (ρ). Its SI derived unit is the **Siemens per meter or mho per meter,**

$$\therefore \qquad \sigma = \frac{1}{\rho} \tag{6.14}$$

Equation (6.14) becomes,

$$\frac{I}{A} = \sigma E \tag{6.15}$$

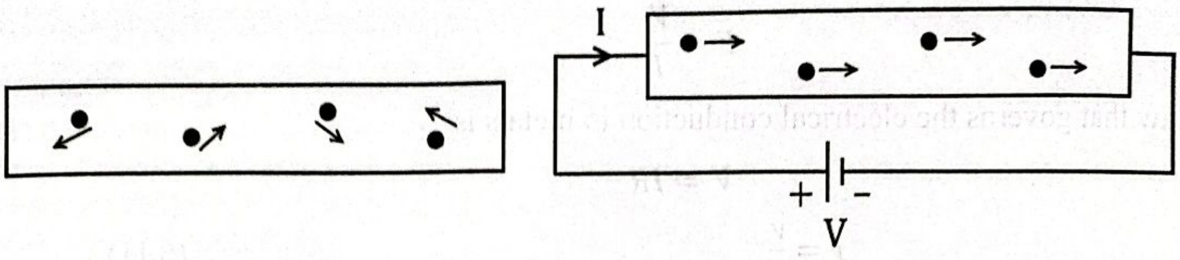
The current density J is defined as the current per unit area of the cross-section of the conductor.

$$J = \frac{I}{A} = \sigma E$$

$$J = \sigma E \tag{6.16}$$

The above equation is known as microscopic form of ohm's law. It describes how voltage, current and resistance are interrelated on a 'microscopic' level.

In absence of an external electric field the free electrons move randomly in all directions. When an electric field is applied the random motion of electrons becomes unidirectional as shown in Fig.(6.5 b).



(a) Random motion of electrons (b) Uni-directional motion of electrons

Fig. 6.5

The average velocity acquired by an electron is directional proportional to applied electric field and is known as ***drift velocity*** v_d.

Thus $$v_d \propto E$$

$$v_d = \mu E \qquad (6.17)$$

where μ is the proportionality constant known as **mobility** of an electron. It is expressed in m^2/volt –sec or cm^2/volt –sec.

The conductivity of a material can be related to the number of charge carriers present in the material.

$$\boldsymbol{I = neAv_d} \qquad (6.18)$$

where v_d is the drift velocity of the electrons.

From equation (6.18) we can write,

$$\frac{I}{A} = nev_d$$

$$J = nev_d \qquad (6.19)$$

Putting $J = \sigma E$ in equation (6.19)

$$\sigma E = nev_d$$

$$\therefore \quad \sigma = ne\frac{v_d}{E}$$

$$\therefore \quad \sigma = n\,e\,\mu \qquad (6.20)$$

Equation (6.20) gives **conductivity** of the conductors.

6.3.2 ELECTRICAL CONDUCTIVITY OF SEMICONDUCTORS

In semiconductors electrons as well as holes are the charge carriers. The current flow is due to the movement of electrons and holes in opposite directions.

The conductivity σ_n of the semiconductor due to electrons in the conduction band is given by

$$\sigma_n = n\,e\,\mu_e \qquad (6.21)$$

where n is the electron density in conduction band per unit volume, e the electron charge and μ_e the electron mobility.

Similarly, the conductivity σ_p of the semiconductor due to holes in the valence band is given by

$$\sigma_p = p\, e\, \mu_h \qquad (6.22)$$

where p is the positive hole density in valence band per unit volume, e the charge on the hole and μ_h the hole mobility.

The total conductivity σ of the semiconductor is given by,

$$\begin{aligned} \sigma &= \sigma_n + \sigma_p \\ &= n\, e\, \mu_e + p\, e\, \mu_h \\ &= e[n\ \mu_e + p\ \mu_h] \qquad (6.23) \end{aligned}$$

Conductivity of intrinsic semiconductors: In intrinsic semiconductors there are equal number of electrons and holes.

$$n = p = n_i \qquad (6.24)$$

where n_i is the intrinsic concentration of electrons or holes in the semiconductor.

Thus the conductivity σ_i of intrinsic semiconductor is given by

$$\begin{aligned} \sigma_i &= e[n_i\ \mu_e + n_i\ \mu_h] \\ &= en_i[\ \mu_e +\ \mu_h] \qquad (6.25) \end{aligned}$$

Conductivity of *n*-type semiconductors: In case of *n*-type semiconductor the electrons are the majority charge carriers i.e. the electron concentration is far greater than the hole concentration ($n \gg p$).

The electron concentration may be represented by n_d, the donor concentration.

From equation (6.25) the conductivity σ_n of n-type semiconductor is given by

$$\sigma_n = n_d\, e\, \mu_e \qquad (6.26)$$

Conductivity of *p*-type semiconductors-: In case of *p*-type semiconductor the holes are the majority charge carriers i.e. the hole concentration is far greater than the electron concentration ($p \gg n$). The hole concentration may be represented by n_a, the acceptor concentration.

From equation (6.26) the conductivity σ_p of *p*-type semiconductor is given by

$$\sigma_p = n_a\, e\, \mu_h \qquad (6.27)$$

6.4 FERMI- DIRAC DISTRIBUTION FUNCTION

There are three types of statistical distributions.

1. **Maxwell-Bolzmann distribution** given by - $f(E) = \dfrac{1}{e^{\left[\frac{(E-\mu)}{kT}\right]}}$

This is classical statistics applies to particles which are identical, distinguishable and having no consideration for the spin. Examples: atoms and molecules.

2. **Bose- Einstein distribution** given by - $f(E) = \frac{1}{\left\{e^{\left[\frac{(E-\mu)}{kT}\right]}-1\right\}}$

This is Quantum statistics applies to particles which are identical, indistinguishable and have integral spin (spin 1). Such particles are known as Bosons. Examples: photons

3. **Fermi-Dirac distribution** given by - $f(E) = \frac{1}{\left\{e^{\left[\frac{(E-\mu)}{kT}\right]}+1\right\}}$

This is also Quantum statistics applies to particles which are identical, indistinguishable and have half-integral spin (spin 1/2). Such particles are known as Fermions. Examples: conduction electrons in metals, semiconductors and insulators.

In Fermi-Dirac statistics *f(E)* represents the probability that if an electron is dropped in an empty box having large number of energy levels it will occupy a particular state with energy *E*.

Fermi-Dirac distribution is mostly written as $f(E) = \frac{1}{\left\{e^{\left[\frac{(E-E_F)}{kT}\right]}+1\right\}}$ (6.28)

where E_F is called Fermi energy and

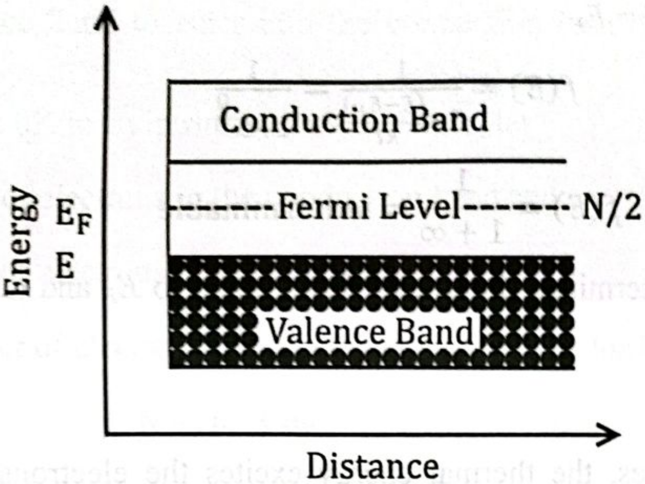
k the Boltzmann constant whose value is 1.38054×10^{-23} joule per kelvin.

6.4.1. FERMI ENERGY LEVEL IN CONDUCTOR

In conductors (metals) the valance band and conduction band overlap each other. In other words, in a conductor there is only a conduction band. This conduction band has more energy levels than the electrons.

(i) At T= 0°K

Let us now consider distribution of the conduction electrons among the various allowed energy levels at absolute zero temperature taking into consideration Pauli's exclusion principle according to which an energy level can accommodate at most two electrons, one with spin up and the other with spin down. Thus in filling the energy levels in the conduction band, two electrons occupy the lowest energy level, two more the next level and so on until all the electrons in the metal have been accommodated. The energy of the highest occupied level is called the ***Fermi energy or Fermi level E_F***. Hence Fermi level may be regarded as the uppermost occupied energy level in a conductor at 0° K.



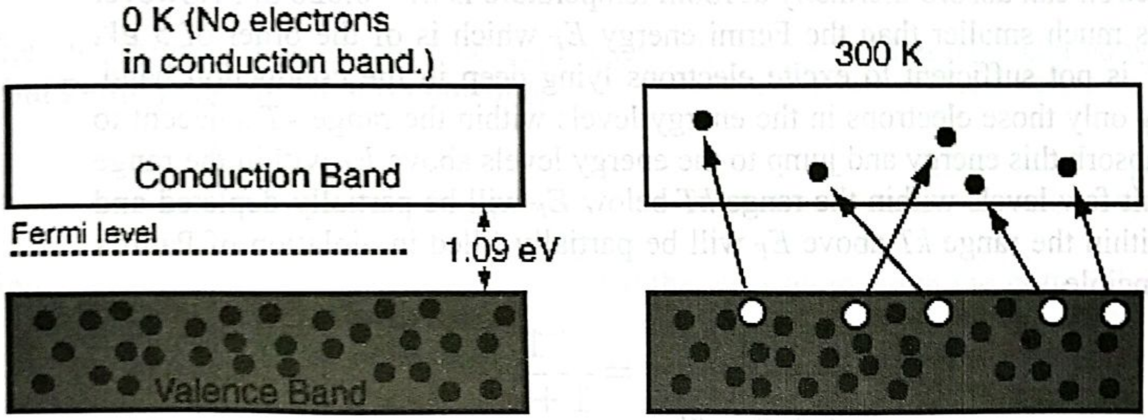


Fig. 6.6 Energy Band Structure of a Conductor at 0^0 K

a) **At $T = 0°$ K,** for energy levels E lying below E_F. Since $E < E_F$, $(E-E_F)$ is a negative quantity. The argument of the exponential function will be $-\infty$. The Fermi Dirac- distribution function thus becomes,

$$f(E) = \frac{1}{1+e^{\frac{(E-E_F)}{kT}}} = \frac{1}{1+e^{-\infty}}$$

$$\therefore \quad f(E) = \frac{1}{1+0} = 1$$

$f(E) = 1$ indicates that all the energy levels below Fermi level are occupied by electrons.

b) **At $T = 0°$ K,** for energy levels E lying above E_F. Since $E > E_F$, $(E-E_F)$ is a positive quantity. The argument of the exponential function will be $+\infty$. The Fermi Dirac- distribution function thus becomes,

$$f(E) = \frac{1}{1+e^{\frac{(E-E_F)}{kT}}} = \frac{1}{1+e^{\infty}}$$

$$\therefore \quad f(E) = \frac{1}{1+\infty} = 0$$

$f(E) = 0$ indicates that all the levels above Fermi level are vacant.

c) **At $T = 0^\circ K$**, for $E = E_F$

$$f(E) = \frac{1}{1+\frac{(E-E_F)}{kT}} = \frac{1}{1+e^{\frac{0}{0}}}$$

$$\therefore \quad f(E) = \frac{1}{1+\infty} = \text{indetrminable}$$

f (E) becomes indeterminate for energy level equal to E_F and ranges between zero and one.

(ii) At $T > 0^\circ K$

As the temperature increases, the thermal energy excites the electrons. The energy which an electron can absorb thermally at room temperature is kT= 0.026 *eV*. However this energy is much smaller than the Fermi energy E_F which is of the order of 5 *eV*. Therefore kT is not sufficient to excite electrons lying deep in the conduction band. Consequently only those electrons in the energy levels within the range kT adjacent to Fermi level absorb this energy and jump to the energy levels above E_F within the range kT. As a result few levels within the range kT below E_F will be partially depleted and few levels within the range kT above E_F will be partially filled in violation of Pauli's exclusion principle.

$$f(E) = \frac{1}{1+e^{\frac{(E-E_F)}{kT}}} = \frac{1}{1+e^{0}}$$

$$\therefore \quad f(E) = \frac{1}{1+1} = \frac{1}{2}$$

It indicates that the probability of electron occupancy of Fermi level at any temperature other than 0°K is 50%.

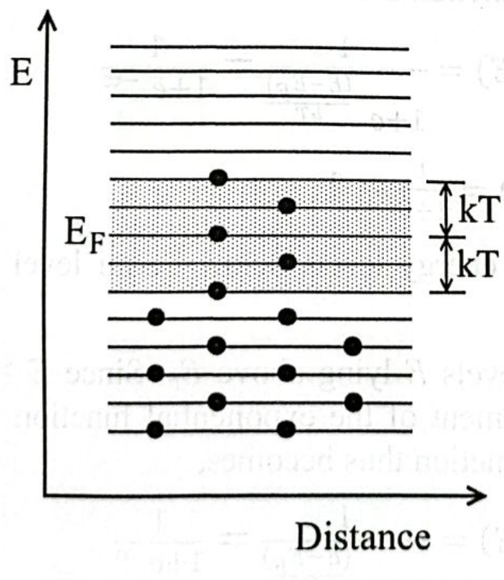


Fig. 6.7 Energy Band Structure of a Conductor at $T > 0^\circ K$

6.4.2. Position of Fermi Level in Intrinsic Semiconductors

In a semiconductor, at 0K the valance band is completely filled and conduction band is completely empty. At room temperature, the thermal excitation causes some of the

electrons from valence band to enter into the conduction band leaving holes in the valance band.

At temperature above 0K in an intrinsic semiconductor, let

n_C – number of electrons in the conduction band and

n_V – number of electrons in valence band.

Therefore total number of electrons in the intrinsic semiconductor is

$$N = n_C + n_V \tag{6.29}$$

Let $f(E_C)$ is the probability that an electron occupies energy E_C in conduction band.

The probability that an electron occupies energy E_C in conduction band can be written from Fermi Dirac distribution function,

$$f(E_C) = \frac{1}{1+e^{\left(\frac{E_C-E_F}{kT}\right)}} \tag{6.30}$$

Here E_C is the minimum energy required for the electron to reach the bottom level of conduction band. The extra energy is converted into its kinetic energy with which the electron moves freely in conduction band at any energy level.

Therefore, the number of electrons occupies energy E_C in conduction band is given by,

$$n_C = N \, . \, f(E_C) \tag{6.31}$$

Substituting equation (6.30) in Equation (6.31)

$$n_C = \frac{N}{1+e^{\left(\frac{E_C-E_F}{kT}\right)}} \tag{6.32}$$

Similarly, the probability that an electron occupies energy E_V in valence band is given by,

$$f(E_V) = \frac{N}{1+e^{-\left(\frac{E_F-E_V}{kT}\right)}} \qquad [(E_V - E_F) \text{ is negative}] \tag{6.33}$$

Therefore, the number of electrons occupies energy E_V in valence band can be written as,

$$n_V = N \, . \, f(E_V) \tag{6.34}$$

Substituting equation (6.33) in Equation (6.34)

$$n_V = \frac{N}{1+e^{-\left(\frac{E_F-E_V}{kT}\right)}} \tag{6.35}$$

Substituting equations (6.32) and (6.35) in Equation (6.29)

$$N = \frac{N}{1 + e^{\left(\frac{E_C - E_F}{kT}\right)}} + \frac{N}{1 + e^{-\left(\frac{E_F - E_V}{kT}\right)}}$$

$$1 = \frac{1}{1 + e^{\left(\frac{E_C - E_F}{kT}\right)}} + \frac{1}{1 + e^{-\left(\frac{E_F - E_V}{kT}\right)}}$$

$$1 = \frac{2 + e^{-\left(\frac{E_F - E_V}{kT}\right)} + e^{\left(\frac{E_C - E_F}{kT}\right)}}{\left[1 + e^{\left(\frac{E_C - E_F}{kT}\right)}\right] \cdot \left[1 + e^{-\left(\frac{E_F - E_V}{kT}\right)}\right]}$$

$$\left[1 + e^{\left(\frac{E_C - E_F}{kT}\right)}\right] \cdot \left[1 + e^{-\left(\frac{E_F - E_V}{kT}\right)}\right] = 2 + e^{-\left(\frac{E_F - E_V}{kT}\right)} + e^{\left(\frac{E_C - E_F}{kT}\right)}$$

$$1 + e^{\left(\frac{E_C - E_F}{kT}\right)} + e^{-\left(\frac{E_F - E_V}{kT}\right)} + e^{\left(\frac{E_C - 2E_F + E_V}{kT}\right)} = 2 + e^{-\left(\frac{E_F - E_V}{kT}\right)} + e^{\left(\frac{E_C - E_F}{kT}\right)}$$

$$e^{\left(\frac{E_C - 2E_F + E_V}{kT}\right)} = 1$$

By logarithm on both sides

$$\frac{E_C - 2E_F + E_V}{kT} = 0$$

$$E_C - 2E_F + E_V = 0$$

$$E_F = \frac{E_C + E_V}{2} \tag{6.35}$$

Equation (6.35) shows that for an intrinsic semiconductor the Fermi level lies at the middle of the forbidden energy gap shown graphically in Fig 6.8.

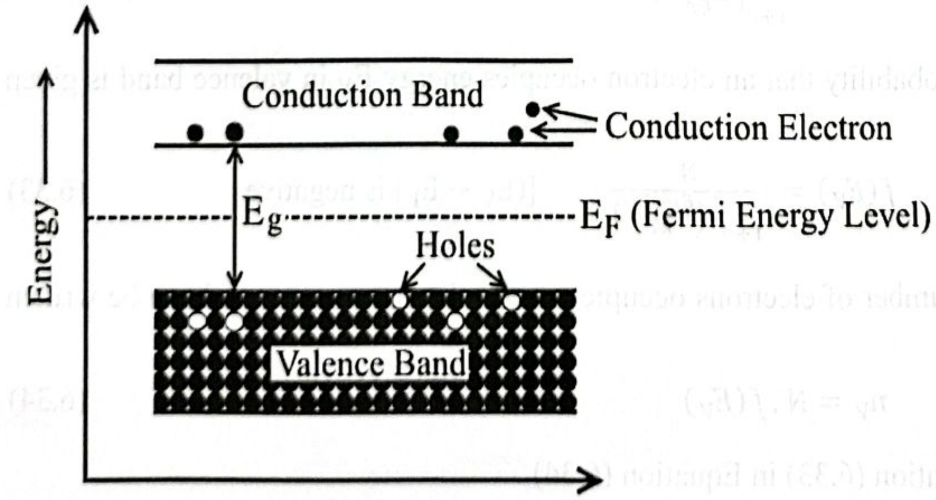


Fig. 6.8 Energy band level diagram of intrinsic semiconductor

6.4.3. FERMI ENERGY LEVEL IN N-TYPE SEMICONDUCTOR

Let the donor impurity atoms of antimony are doped in a silicon crystal. When the

density of donor atoms is low they are distantly spaced from one another, approximately by 100 atom spacing. Since the donor atoms are distinctly spaced, they cannot interact with each other and there is no formation of donor energy band. Their energy levels are shown by a single energy level. The amount of energy required to detach the electron completely from the normal atom is called the ionization energy. The ionization energy of the impurity atom in *Si* about 0.05 *eV* (in *Ge* it is 0.01 *eV*). Thus the impurity states of the impurity atom lie within the forbidden energy gap, specifically a few hundredths of an electron volt below the conduction band.

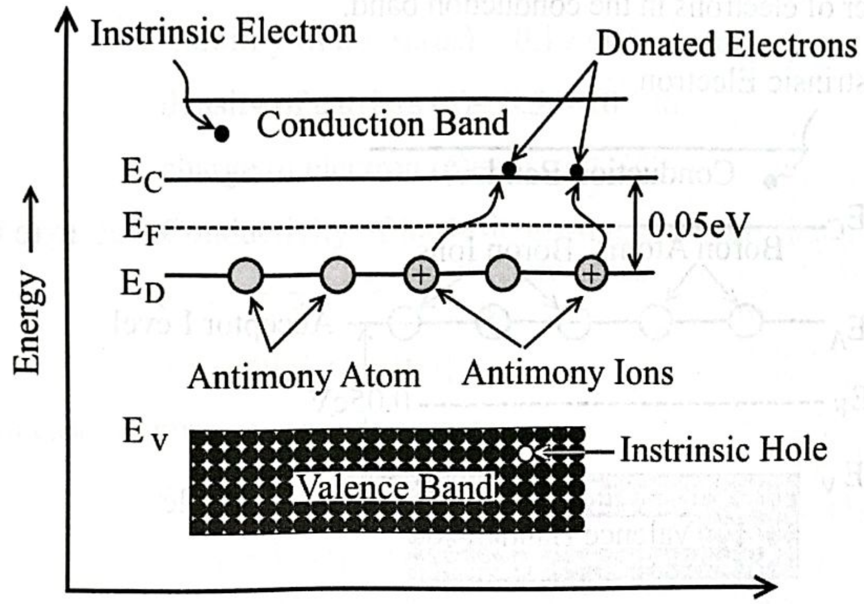


Fig. 6.9 Energy level diagram of *n*-type semiconductor

At 0°K, the conduction band is empty because the donor atoms are not ionized. If the temperature is raised so as to supply only 0.05*eV* energy these atoms will ionize and donate electrons to the conduction band. Thus each antimony atom donates one electron to conduction band without producing hole in valence band. The antimony atoms become ions. Even though the ions are charged they are immobile and do not contribute to the current.

At ordinary temperatures the intrinsic process produces some electrons and equal number of holes in the valence band. The electrons jump to the conduction band. This is the second source of electrons made available for conduction. Thus the electrons outnumber the holes and are the majority charge carriers while holes are minority charge carriers in *n*-type semiconductor.

6.4.4. FERMI ENERGY LEVEL IN P-TYPE SEMICONDUCTOR

Let the acceptor impurity atoms of boron are doped in a silicon crystal. When the density of acceptor atoms is low they are distantly spaced from one another, approximately by 100 atom spacing. Since the acceptor atoms are distinctly spaced, they cannot interact with each other and there is no formation of acceptor energy band. Their energy levels are shown by a single energy level.The energy involved in capturing electrons from neighbouring silicon atoms is very small. It is about 0.05 *eV* in silicon and 0.01 *eV* in

germanium. Hence the energy level of acceptor atoms lies 0.05 *eV* above the valence band for silicon (0.01 *eV* above the valence band in case of germanium).As temperature is raised the impurity atoms accept electrons from the valence band. These exited electrons leave behind holes in the valence band. Thus the holes are produced in the valence band without giving electrons to the conduction band.

The boron atoms become ions. Even though the ions are charged they are immobile and do not contribute to the hole current. At finite temperatures the holes acquire thermal energy and move downward into the valence band. This intrinsic process produces some holes and equal number of electrons in the conduction band.

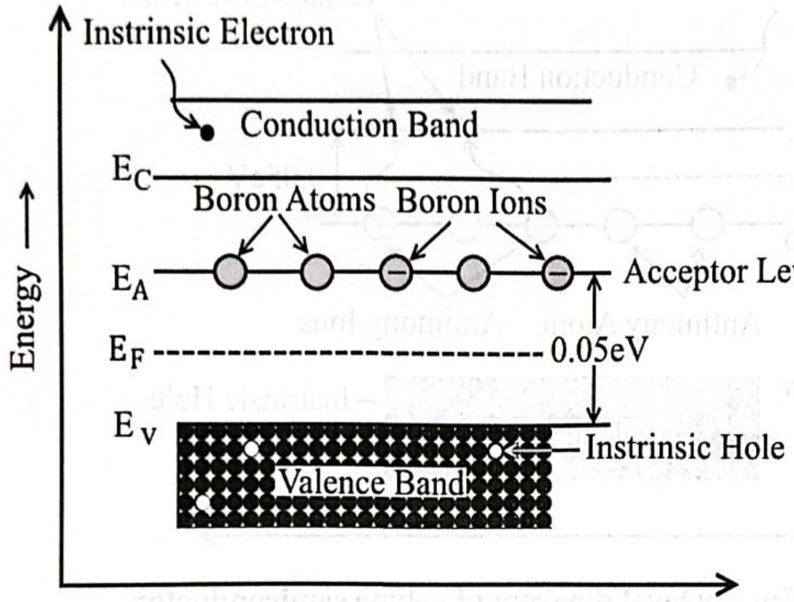


Fig. 6.10 Energy level diagram of *p*-type semiconductor

This is the second source of holes made available for hole conduction. Thus the holes outnumber the electrons and are the majority charge carriers while electrons are minority charge carriers in *p*-type semiconductor.

SOLVED PROBLEMS

1. The resistivity of Cu is 1.72 × 10⁻⁸ ohm- m. Calculate the mobility of electrons in Cu. Given that the number of electrons per unit volume is 10.41 × 10²⁸/ m³.

Solution-: Given resistivity of Cu (ρ) $=1.72 \times 10^{-8}$ ohm- m,

Number of electrons per unit volume (n) $=10.41 \times 10^{28}/\mathrm{m}^3$.

Formula Conductivity

$$\sigma = \frac{1}{\rho} = \frac{1}{1.72 \times 10^{-8}}$$

$$= 58.13 \times 10^{6} \text{ mho per meter}$$

$$\sigma = ne\mu$$

$$\mu = \frac{\sigma}{ne} = \frac{58.13 \times 10^6}{10.41 \times 10^{28} \times 1.6 \times 10^{-19}}$$

Mobility of electrons (μ)= 3.488 × 10^{-3} m^2/volt-sec

2. Find resistivity of intrinsic germanium at 300^0 K. Given density of carriers is $6.5 \times 10^{19}/m^3$. Mobility of electrons is 0.39 m^2/ volt-sec and mobility of holes is 0.19 m^2/v-sec. charge of electron is 1.6×10^{-19} C.

Solution-: **Given** Mobility of electrons (μ_e) =0.39 m^2/volt-sec

Mobility of holes (μ_h) = 0.19 m^2/volt-sec

density of carriers (n)= $6.5 \times 10^{19}/m^3$

charge of electron (e)=1.6×10^{-19} C.

Formula Conductivity of intrinsic germanium is given by, σ

$$\sigma_{total} = \sigma_n + \sigma_p$$

$$= [\,(ne\mu_n) + (pe\mu_h)]$$

In case of intrinsic semiconductor

$$n = p = 6.5 \times 10^{19} /m^3$$

$$\sigma_{total} = [ne\,(\mu_n + \mu_h)]$$

$$\sigma_{total} = [6.5 \times 10^{19} \times 1.6 \times 10^{-19}\,(0.39 + 0.19)]$$

$6.5 \times 1.6 \times 0.53 = 6.32$ mho/meter.

Conductivity of intrinsic germanium (**σ_{total}**) = **6.32 mho/meter**

∴ resistivity of intrinsic germanium (ρ)

$$\rho = \frac{1}{\sigma} = \frac{1}{6.32}$$

3. What is the probability of an electron being thermally excited to conduction band in silicon at 20^o C if the band gap energy is 1.12eV? Given Boltzmann constant 1.38×10^{23} J/K.

Solution-: **Given:** band gap energy (E_g) = 1.12eV

Boltzmann constant (k) = 1.38×10^{23} J/K.

$$= \frac{1.38 \times 10^{23}}{1.6 \times 10^{-19}}$$

Boltzmann constant (k) in eV = 8.625×10^5 eV/K

$T = 20^0$ C = 273 +20 =293^0 K

Formula: The probability of an electron being thermally excited to conduction band is given by,

$$f(E_C) = \frac{1}{1+e^{\frac{(E_C-E_V)}{kT}}}$$

For intrinsic semiconductor,

$$(E_C - E_V) = \frac{E_g}{2} = \frac{1.12}{2} = 0.56 \text{ eV}$$

$$f(E_C) = \frac{1}{1+e^{\left(\frac{0.56}{8.625\times10^5\times293}\right)}}$$

$$f(E_C) = \frac{1}{1+e^{(26.1595\times10^{-9})}}$$

$$f(E_C) = 6.37821 \times 10^{-10}$$

4. In a solid, consider the energy level lying 0.012 eV below Fermi level. What is the probability of this level not occupied by an atom?

Solution:

Given: $E_F - E = 0.012$ eV

Boltzmann's constant (k) $=1.38 \times 10^{-23}$ J/K,

$$k = \frac{1.38 \times 10^{-23}}{1.6 \times 10^{-19}} = 8.625 \times 10^{-5}\ eV/K$$

Temperature (T) = 300 K

We have,

$$f(E) = \frac{1}{1+e^{\left(\frac{E-E_F}{kT}\right)}}$$

The probability of energy level not being occupied $= 1 - f(E)$

$$= 1 - \frac{1}{1+e^{\left(\frac{E-E_F}{kT}\right)}}$$

$$= 1 - \frac{1}{1+e^{\left(\frac{0.012}{8.625\times10^{-5}\times300}\right)}} = 1 - 0.614$$

The probability of energy level not being occupied = 0.386

5. Fermi energy for silver is 5.5 eV. Find out the energy for which the probability of occupancy at 300 K is 0.9.

Solution:

Given: Fermi energy for silver (E_F) = 5.5 eV

Boltzmann's constant (k) = 1.38×10^{-23} J/K,

$$k = \frac{1.38 \times 10^{-23}}{1.6 \times 10^{-19}} = 8.625 \times 10^{-5}\, eV/K$$

Temperature (T) = 300 K

$$f(E) = \frac{1}{1 + e^{\left(\frac{E-E_F}{kT}\right)}}$$

$$0.9 = \frac{1}{1 + e^{\left(\frac{E-5.5}{8.625 \times 10^{-5} \times 300}\right)}}$$

$$\frac{1}{0.9} = 1 + e^{\left(\frac{E-5.5}{8.625 \times 10^{-5} \times 300}\right)}$$

$$1.1111 = 1 + e^{\left(\frac{E-5.5}{8.625 \times 10^{-5} \times 300}\right)}$$

$$0.1111 = e^{\left(\frac{E-5.5}{8.625 \times 10^{-5} \times 300}\right)}$$

By taking logarithm

$$-6.1973 = \frac{E - 5.5}{8.625 \times 10^{-5} \times 300}$$

$$-0.056855 = E - 5.5$$

$$E = 5.44 \text{ eV}$$

6. For intrinsic silicon, the room temperature electrical conductivity is 4×10^{-4} mho per meter. The electron and hole mobilities are 0.14 and 0.040 m^2/volt-sec respectively. Compute the electron and hole concentration at room temperature.

Solution: **Given:** Electrical conductivity (σ) = 4×10^{-4} mho per meter,

Mobility of Electron, $\mu_e = 0.14\ m^2$/volt- sec,

Mobility of holes, $\mu_h = 0.040\ m^2$/volt- sec

Formula $\sigma_i = n_i e (\mu_e + \mu_h)$

$$n_i = \frac{\sigma_i}{e(\mu_e + \mu_h)}$$

$$= \frac{4 \times 10^{-4}}{1.6 \times 10^{-19}(0.14 + 0.040)} = \frac{4 \times 10^{-4}}{6.88 \times 10^{-20}}$$

$\therefore n_i = 1.388 \times 10^{16}\ m^{-3}$

7. The intrinsic carrier density at room temperature in Ge is $6.37 \times 10^{19}/m^3$. If the electron and hole mobilities are 0.38 and 0.18 m^2 /volt- sec respectively. Calculate its resistivity.

Solution: **Given:** Intrinsic carrier density (n_i) = $6.37 \times 10^{19}/m^3$

mobility of electron (μ_e) = 0.38 m^2 /volt- sec

mobility of hole(μ_h) = 0.18 m^2 /volt- sec

resistivity(ρ) = ?

Formula Conductivity of intrinsic semiconductor is given by,

$$\sigma = n_i e (\mu_e + \mu_h) \qquad (\sigma = \frac{1}{\rho})$$

$$\rho = \frac{1}{n_i\, e\, (\mu_e + \mu_h)}$$

$$= \frac{1}{6.37\times10^{19}\times1.6\times10^{-19}\ (0.38 + 0.18)}$$

$$= \frac{1}{6.1232}$$

$\therefore$ **resistivity (ρ) = 0.4719 Ω- m**

8. Determine the concentration of conduction electrons in a sample of silicon, if one in every million silicon atom is replaced by a phosphorus atom. Assume every phosphorus atom to be singly ionised. Silicon has a molar mass of 0.028 kg/mole and density of 2300 kg/m^3.

Solution:

Given: Molar mass of silicon = 0.028 kg/mole, density of silicon =2300 kg/m^3

$$\text{Concentration of silicon atoms} = \frac{\text{density of silicon}}{\left(\frac{\text{molar mass of silicon}}{\text{Avagadro's number}}\right)}$$

$$= \frac{\text{density of silicon}}{\text{molar mass of silicon}} \times \text{Avagadro's number}$$

$$= \frac{2300}{0.028} \times 6.023 \times 10^{26}$$

$$\text{Concentration of silicon atom} = 4.947 \times 10^{31}/m^3$$

One phosphorus atom replaces one silicon atom in every million silicon (10^6) atoms.

$$\therefore \text{number of donor atoms (phosphrous atoms)} = \frac{4.947 \times 10^{31}}{10^6}$$

$$= 4.947 \times 10^{25}\ atoms$$

Every phosphorus atom to be singly ionised

∴ The concentration of conduction electron in a sample of silicon $= 4.947 \times 10^{25}$ atoms

9. Find the resistance of an intrinsic Ge rod 1 cm long. 1mm wide and 1mm thick at 300 K. for Ge $n_i = 6.5 \times 10^{19}/m^3$, $\mu_e = 0.39\ m^2$/volt sec, and $\mu_h = 0.19\ m^2$/volt- sec.

Solution-: Given: Length of intrinsic Ge rod (l) = 1 cm =1×10^{-2} m,

Breadth (w) =1mm =1×10^{-3} m, Thickness=1×10^{-3} m

Mobility of electrons and holes $\mu_e = 0.39\ m^2$/volt sec,

and $\mu_h = 0.19\ m^2$/volt sec.

Formula $\sigma_i = n_i e (\mu_e + \mu_h)$

$= 6.5 \times 10^{19} \times 1.6 \times 10^{-19} (0.39 + 0.19)$

$= 6.5 \times 10^{19} \times 1.6 \times 10^{-19} \times 0.58$

Conductivity (σ_i) = 6.32 mho per meter

Resistance of an intrinsic semiconductor is given by,

$$R = \frac{\rho l}{A} = \frac{l}{\sigma A} \quad \left(\rho = \frac{1}{\sigma}\right)$$

$$R = \frac{1 \times 10^{-2}}{6.32\times(1\times10^{-3}\times1\times10^{-3})} \quad [\therefore \text{Area(A)= Breadth} \times \text{thickness}]$$

$$R = \frac{1 \times 10^{-2}}{6.32\times\times10^{-6}}$$

$R = 4310\ \Omega$

10. Mobility of electrons and holes in a sample of intrinsic germanium at room temperature are 3600 cm^2/volt-sec. and 1700 cm^2/volt-sec. respectively. If the electron and hole densities are equal to 6.5×10^{13} per cm^3. Calculate its conductivity.

Solution-: Given Mobility of electrons (μ_e) = 3600 cm^2/volt-sec =0.36 m^2/volt-sec
Mobility of holes (μ_p) =1700 cm^2/volt-sec = 0.17 m^2/volt-sec

Formula Conductivity of intrinsic germanium is given by, σ

$\sigma_{total} = \sigma_n + \sigma_p$

$= [(ne\mu_n) + (pe\mu_h)]$

In case of intrinsic semiconductor

$n = p = 6.5 \times 10^{13}\ /cm^3 = 6.5 \times 10^{19}\ /m^3$

$\sigma_{total} = [ne\ (\mu_n + \mu_h)]$

$$\therefore \quad \sigma_{total} = [6.5 \times 10^{19} \times 1.6 \times 10^{-19} (0.36 + 0.17)]$$

$$6.5 \times 1.6 \times 0.53 = 6.12 \text{ mho/meter.}$$

Conductivity of intrinsic germanium (σ_{total}) = 6.12 mho/meter

SHORT ANSWER TYPE QUESTIONS

1. What are the advantages of semiconductors?
2. What are intrinsic and extrinsic semiconductors semiconductors?
3. Define mobility of charge carriers and state its SI unit
4. Define the terms direct and indirect energy band gap
5. Explain the concept of Fermi level.
6. Define Fermi energy level. Where is it located in case of intrinsic semiconductors?
7. What is Fermi energy and Fermi- Dirac distribution function?

DESCRIPTIVE ANSWER TYPE QUESTIONS

1. What do you mean by conductivity of a semiconductor? Obtain an expression for the conductivity of Intrinsic, n- type and p- type semiconductors.
2. What is mobility and conductivity of charge carriers? Which has the greater mobility, electron or hole and why?
3. Show that for an intrinsic semiconductor, the Fermi level lies half way between conduction and valence band. With the help of energy band diagram, show the Fermi level positions for n and p- type semiconductors at 0K and TK.
4. Differentiate between direct and indirect band gap semiconductor.
5. What is Fermi energy and Fermi- Dirac distribution function? Show that in intrinsic Semiconductors Fermi level lies midway between Conduction band and valance band.
6. What is Fermi level? Draw suitable diagram to depict its position in intrinsic, p- type, n-type semiconductor and explain it.